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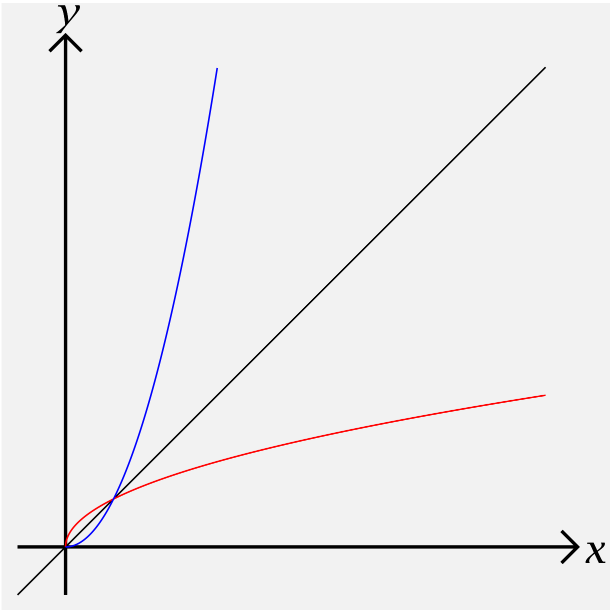

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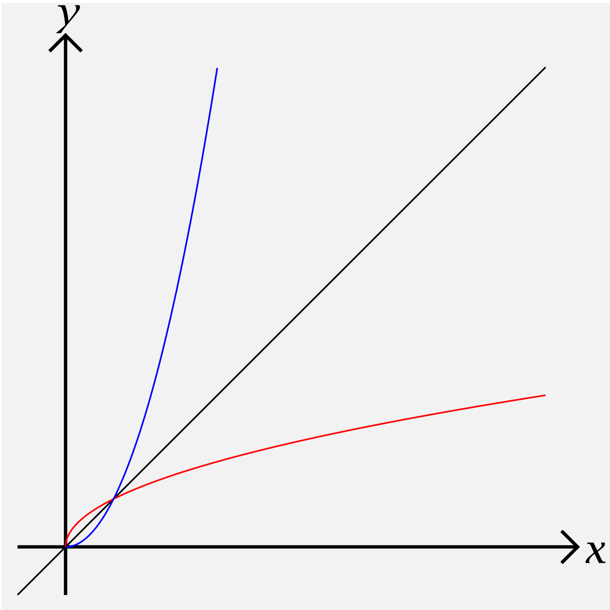

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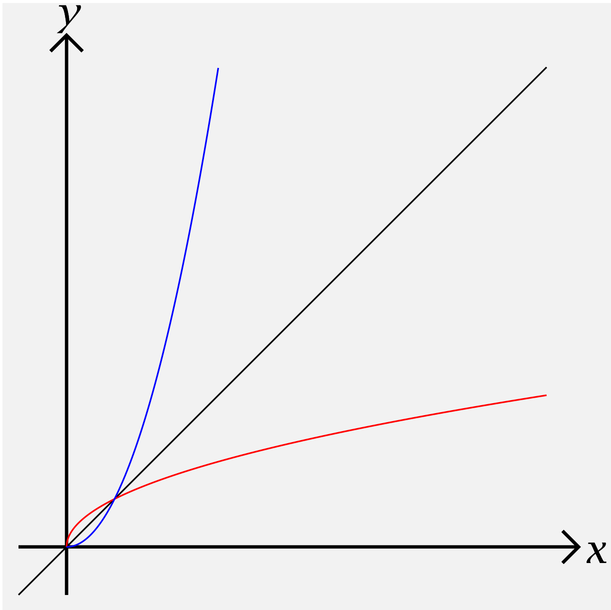

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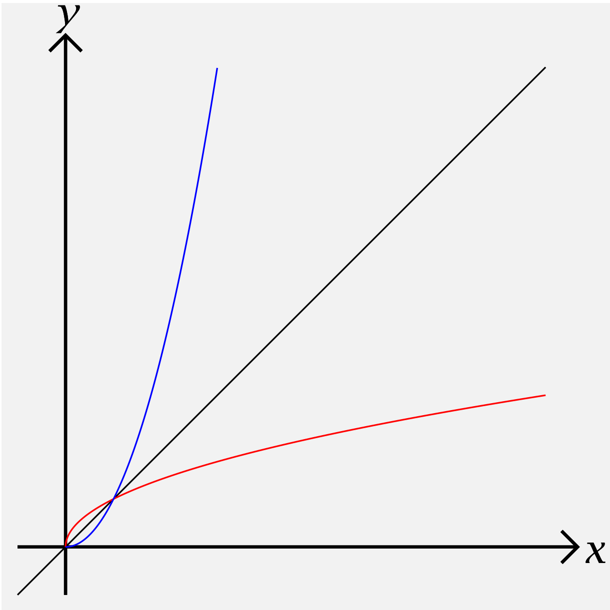

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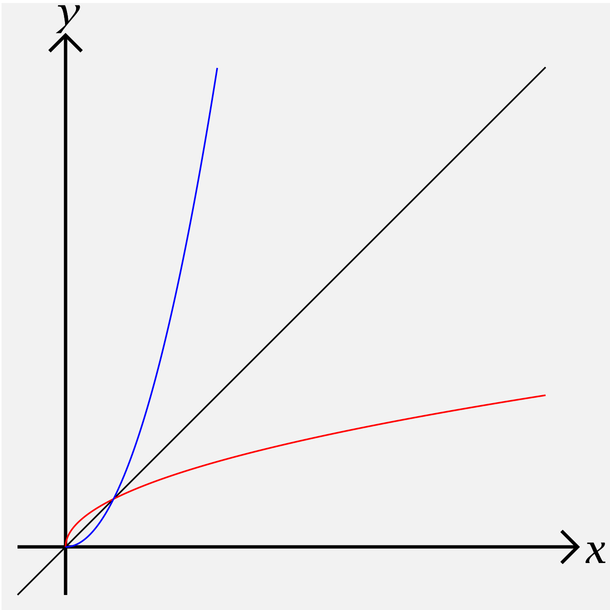

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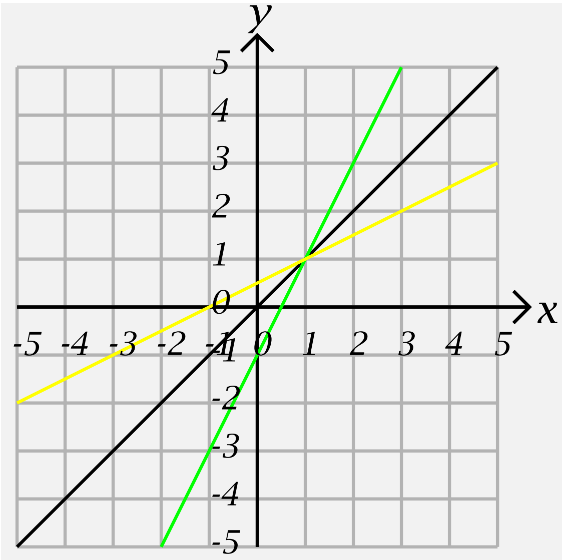

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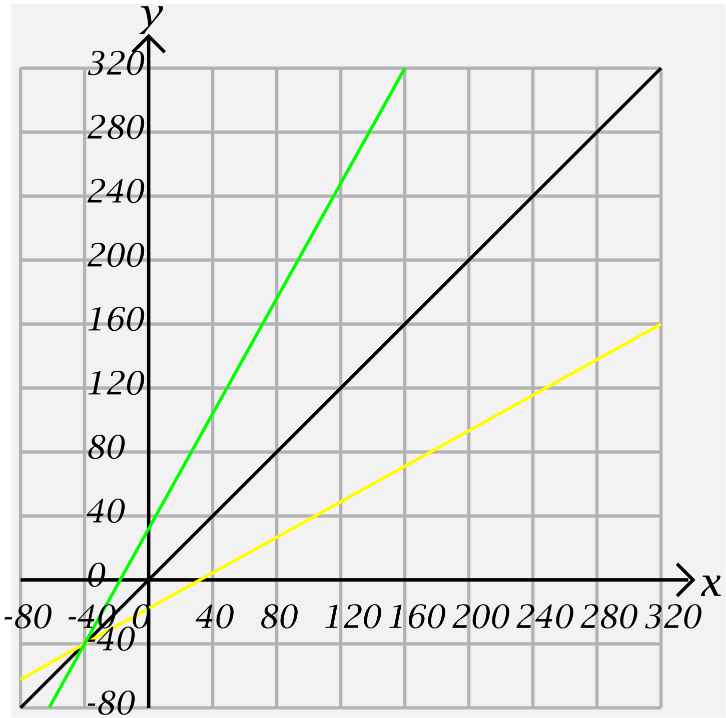
These two formulas are usually written as $C = \frac{5}{9}F - \frac{160}{9}$ and $F = \frac{9}{5}C + 32$, where F and C represent the Fahrenheit and Celsius temperatures respectively.



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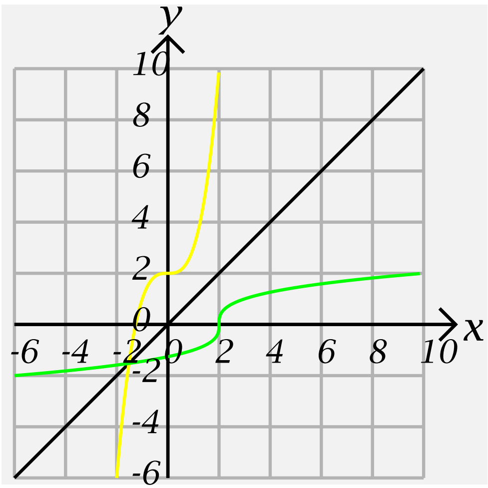

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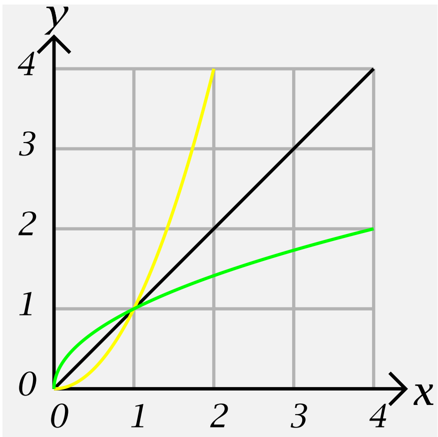
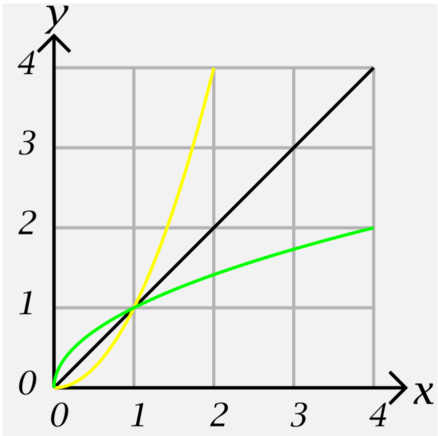


Example 4: $f(x) = x^2$ is 1:1 on the interval $(-\infty, 0]$,

Example 3: $f(x) = x^2$ is 1:1 on the interval $[0, \infty)$, and its inverse is used to define $\sqrt{x}$.



Example 4: $f(x) = x^2$ is 1:1 on the interval $(-\infty, 0]$, and its inverse is $-\sqrt{-x}$.