

Limit Laws

We let \lim stand for any one of the limits $\lim_{x \rightarrow a}$, $\lim_{x \rightarrow a^-}$, $\lim_{x \rightarrow a^+}$, $\lim_{x \rightarrow -\infty}$,
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$$(1) \lim [f(x) + g(x)] = \lim f(x) + \lim g(x)$$

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$$(4) \lim \frac{f(x)}{g(x)} = \frac{\lim f(x)}{\lim g(x)} \text{ if } \lim g(x) \text{ is not equal to } 0.$$

Limit Laws-2

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(8) If c is a constant then $\lim c = c$ for any a .

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$$(11) \lim_{x \rightarrow a} \sqrt[n]{x} = \sqrt[n]{a}, \lim_{x \rightarrow a^-} \sqrt[n]{x} = \sqrt[n]{a}, \lim_{x \rightarrow a^+} \sqrt[n]{x} = \sqrt[n]{a}, \text{ for any}$$

$a > 0$,

if n is a positive integer.

(12) If $f(x) = g(x)$ for all x except possibly $x = a$, then
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(13) If $f(x)$ is a polynomial or a rational function and a is in the domain of f , then $\lim_{x \rightarrow a} f(x) = f(a)$

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$$\lim_{x \rightarrow 0^-} |x| = \lim_{x \rightarrow 0^-} (-x) = (-0) = 0$$

$$\lim_{x \rightarrow 0^+} |x| = \lim_{x \rightarrow 0^+} x = 0$$

Since the left and right hand limits are equal, we can conclude that the two-sided limit does exist and equals the common value 0.

Example 7:

$$\lim_{x \rightarrow 0} \frac{|x|}{x}$$

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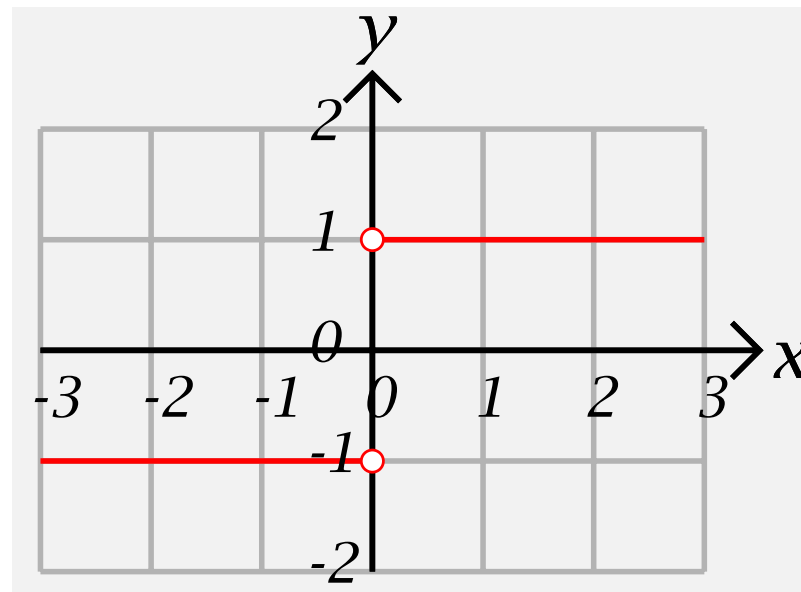
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Note that the limiting value of x , 0, is not in the domain of the function $\frac{|x|}{x}$.

Again, $|x|$ is a function defined by a multicolon formula, and different formulas apply for x on either side of the limiting value of x , so again we must calculate the two one-sided limits:



Limit Laws-10

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Since the left and right hand limits are unequal, we can conclude that the two-sided limit does not exist.

Limits of Rational Functions

If $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials, then

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If $p(a) = 0$, it is possible, but not guaranteed, that $\lim_{x \rightarrow a} f(x)$ can exist.

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where a_0 and b_0 are the leading coefficients of $p(x)$ and $q(x)$.

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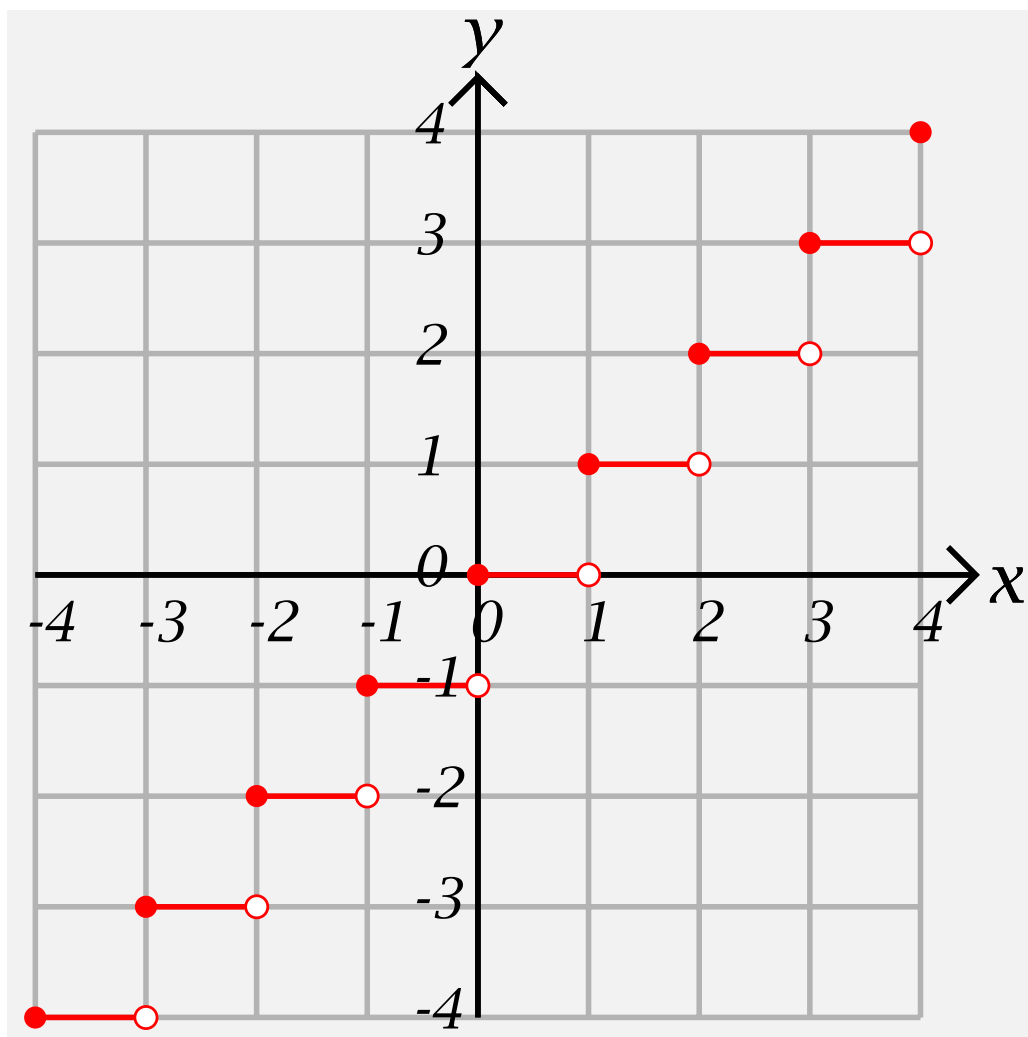
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The Floor Function:



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