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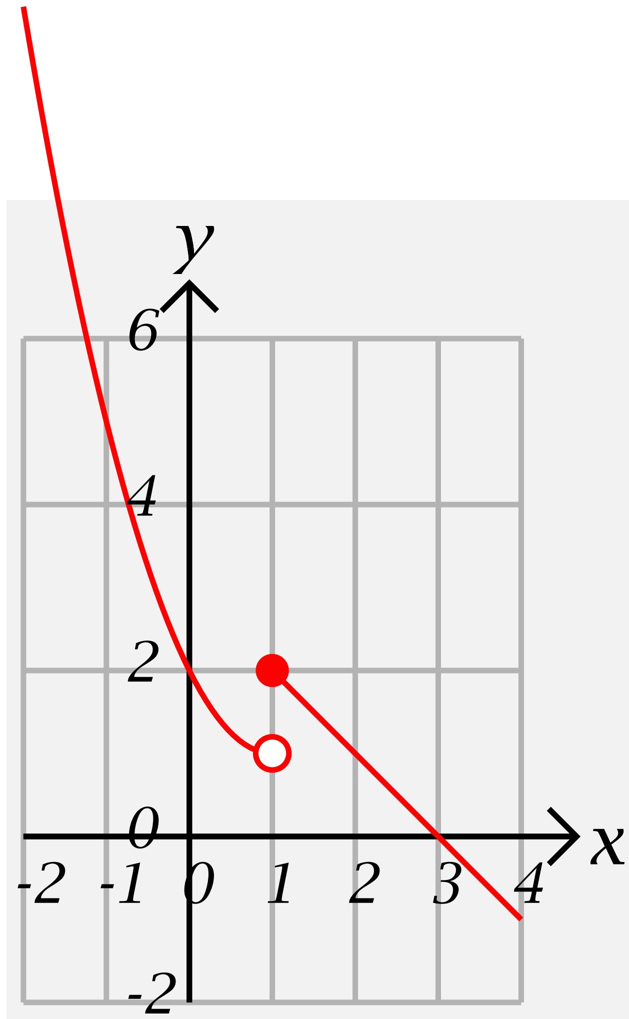

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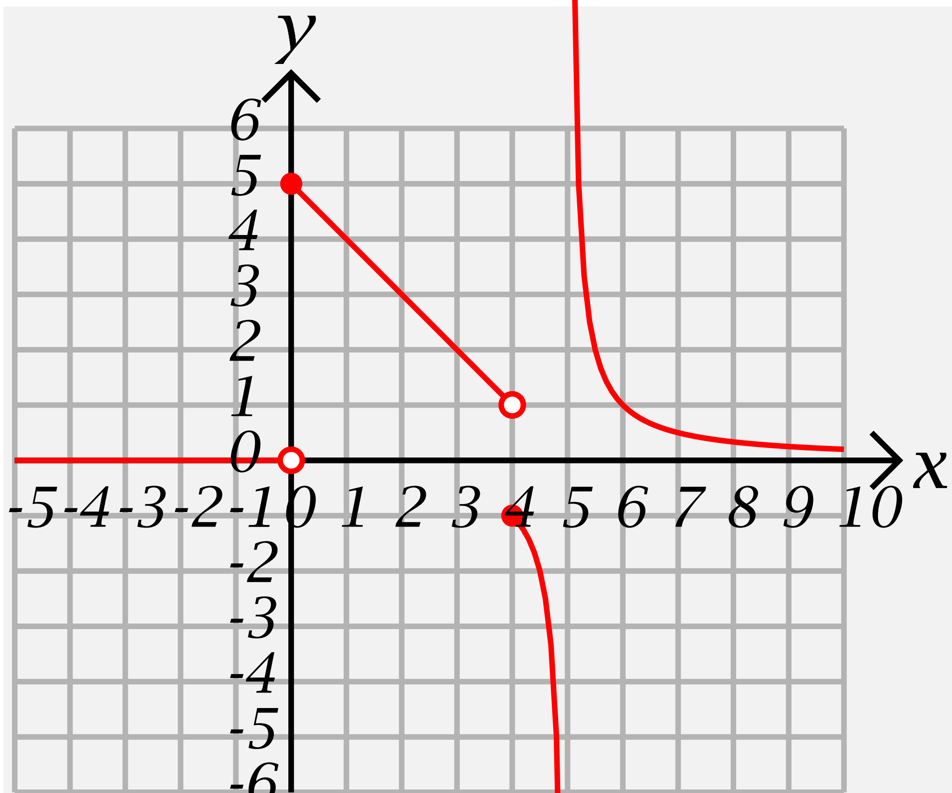

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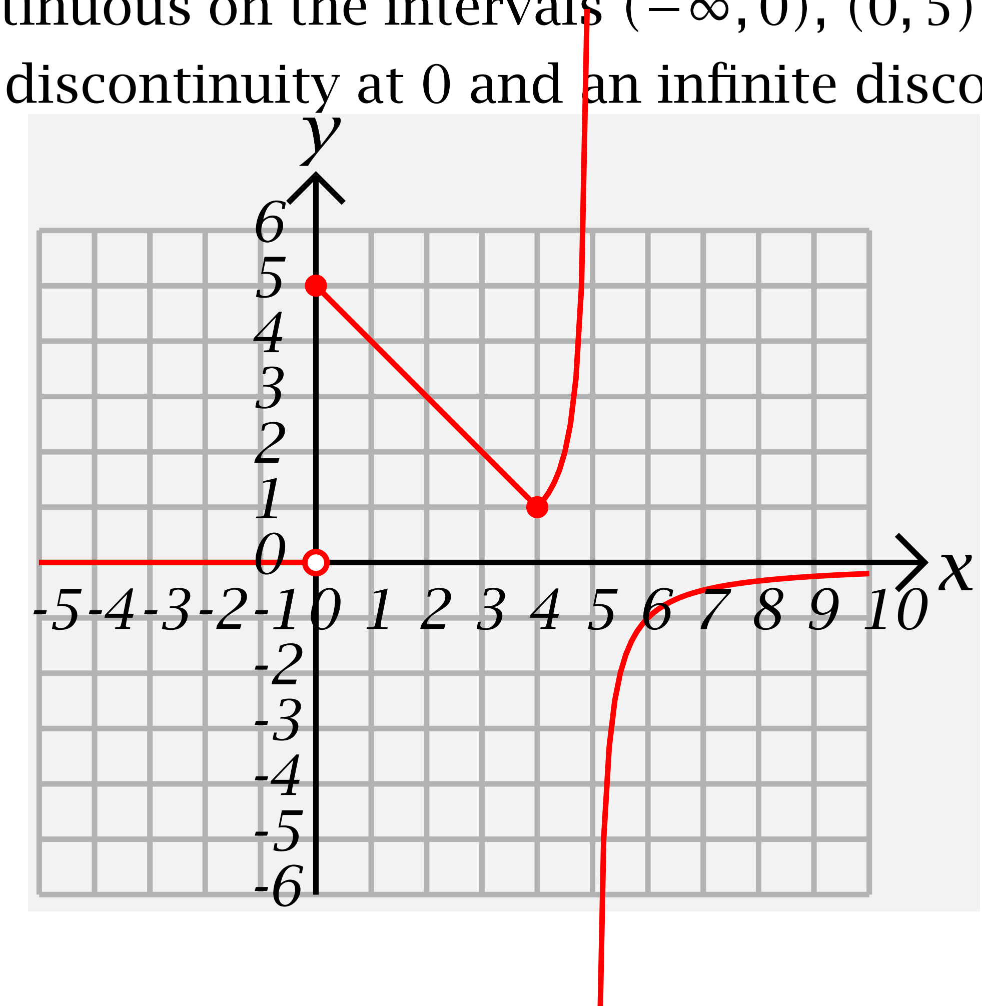

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Definition: A function f is continuous **on a set E** if it is continuous on every open interval contained in E.

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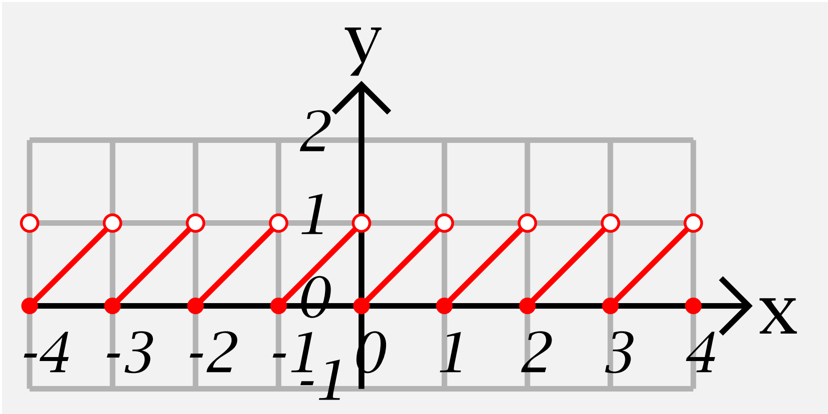

Theorem: If the functions f and g are continuous at a and if c is a constant, then the functions $f + g$, $f - g$, cf, fg, and $\frac{f}{g}$(if $g(a) \neq 0$) are also continuous at a.

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Theorem: Polynomial, rational, root, exponential and logarithmic functions, and algebraic combinations of such functions are continuous on their domains.

Theorem: If f is continuous at b and $\lim_{x \to a} g(x) = b$ then

$$\lim_{x \to a} f(g(x)) = f(\lim_{x \to a} g(x)) = f(b)$$

Theorem: Compositions of continous functions are continuous on their domains.

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