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Example 4:

$$\lim_{x\to 2}\frac{x-2}{x^2-4} =$$

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$$\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9} = \lim_{x\to 9}\left(\frac{\sqrt{x}-3}{x-9}\right)\left(\frac{\sqrt{x}+3}{\sqrt{x}+3}\right) = \lim_{x\to 9}\frac{(\sqrt{x})^2-3^2}{(x-9)(\sqrt{x}+3)} =$$

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$$\lim_{x\to 1}\frac{x^2-1}{x-1}=$$

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Example 6: $\lim_{x\to 0}|x|$

Example 3:

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DANGER! $|x|$ is a function defined by a multicolumn formula, and different formulas apply for $|x|$ on either side of the limiting value of x, namely 0.

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Example 6: $\lim_{x\to 0}|x|$

DANGER! $|x|$ is a function defined by a multicolumn formula, and different formulas apply for $|x|$ on either side of the limiting value of x, namely 0. We must calculate the two one-sided limits:

Example 3:

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$$\lim_{x\to 0^-}|x|=\lim_{x\to 0^-}(-x)=(-0)=0$$

Example 3:

$$\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} = \lim_{x\to 9} \left(\frac{\sqrt{x}-3}{x-9}\right)\left(\frac{\sqrt{x}+3}{\sqrt{x}+3}\right) = \lim_{x\to 9} \frac{(\sqrt{x})^2-3^2}{(x-9)(\sqrt{x}+3)} =$$

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Example 5:

$$\lim_{x\to 1} \frac{x^2-1}{x-1} = \lim_{x\to 1} \frac{(x+1)(x-1)}{x-1} = \lim_{x\to 1}(x+1) = 1+1 = 2$$

Example 6: $\lim_{x\to 0} |x|$

DANGER! $|x|$ is a function defined by a multicolumn formula, and different formulas apply for $|x|$ on either side of the limiting value of x, namely 0. We must calculate the two one-sided limits:

$$\lim_{x\to 0^-} |x| = \lim_{x\to 0^-} (-x) = (-0) = 0$$

$$\lim_{x\to 0^+} |x| = \lim_{x\to 0^+} x = 0$$

Example 3:

$$\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}=\lim_{x\to 9}\left(\frac{\sqrt{x}-3}{x-9}\right)\left(\frac{\sqrt{x}+3}{\sqrt{x}+3}\right)=\lim_{x\to 9}\frac{(\sqrt{x})^2-3^2}{(x-9)(\sqrt{x}+3)}=$$

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Example 5:

$$\lim_{x\to 1}\frac{x^2-1}{x-1}=\lim_{x\to 1}\frac{(x+1)(x-1)}{x-1}=\lim_{x\to 1}(x+1)=1+1=2$$

Example 6: $\lim_{x\to 0}|x|$

DANGER! $|x|$ is a function defined by a multicolumn formula, and different formulas apply for $|x|$ on either side of the limiting value of x, namely 0. We must calculate the two one-sided limits:

$$\lim_{x\to 0^-}|x|=\lim_{x\to 0^-}(-x)=(-0)=0$$

$$\lim_{x\to 0^+}|x|=\lim_{x\to 0^+}x=0$$

Since the left and right hand limits are equal, we can conclude that the two-sided limit does exist and equals the common value 0.

Example 7:

$$\lim_{x \to 0} \frac{|x|}{x}$$

Since the left and right hand limits are equal, we can conclude that the two-sided limit does exist and equals the common value 0.

Example 7:

$$\lim_{x \to 0} \frac{|x|}{x}$$

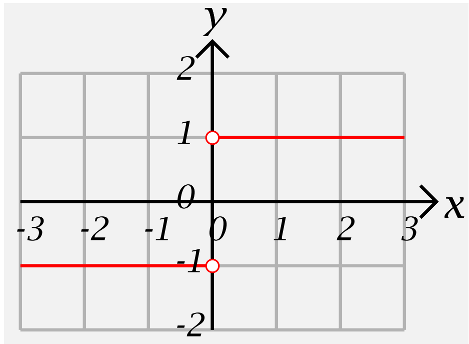
Note that the limiting value of x, 0, is not in the domain of the function $\frac{|x|}{x}$.

Since the left and right hand limits are equal, we can conclude that the two-sided limit does exist and equals the common value 0.

Example 7:

$$\lim_{x \to 0} \frac{|x|}{x}$$

Note that the limiting value of x, 0, is not in the domain of the function $\frac{|x|}{x}$.
Again, $|x|$ is a function defined by a multicolumn formula, and different formulas apply for x on either side of the limiting value of x, so again we must calculate the two one-sided limits:



$$\lim_{x \to 0^-} \frac{|x|}{x} =$$

Since the left and right hand limits are equal, we can conclude that the two-sided limit does exist and equals the common value 0.

Example 7:

$$\lim_{x \to 0} \frac{|x|}{x}$$

Note that the limiting value of x, 0, is not in the domain of the function $\frac{|x|}{x}$.

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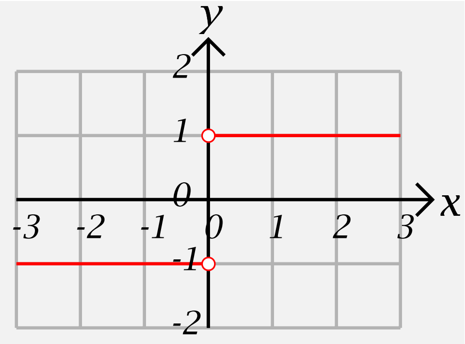

$$\lim_{x \to 0^-} \frac{|x|}{x} = \lim_{x \to 0^-} \frac{-x}{x} =$$

Since the left and right hand limits are equal, we can conclude that the two-sided limit does exist and equals the common value 0.

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$$\lim_{x \to 0} \frac{|x|}{x}$$

Note that the limiting value of x, 0, is not in the domain of the function $\frac{|x|}{x}$.
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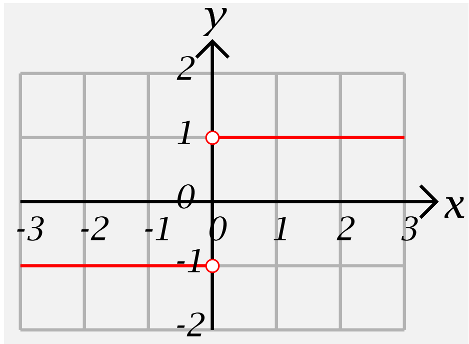

$$\lim_{x \to 0^-} \frac{|x|}{x} = \lim_{x \to 0^-} \frac{-x}{x} = \lim_{x \to 0^-} -1 =$$

Since the left and right hand limits are equal, we can conclude that the two-sided limit does exist and equals the common value 0.

Example 7:

$\lim_{x\to 0} \frac{|x|}{x}$

Note that the limiting value of x, 0, is not in the domain of the function $\frac{|x|}{x}$.
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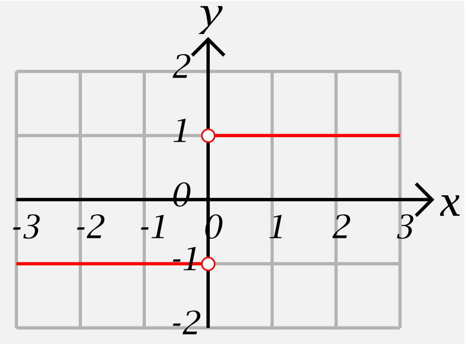

$$\lim_{x\to 0^-} \frac{|x|}{x} = \lim_{x\to 0^-} \frac{-x}{x} = \lim_{x\to 0^-} -1 = -1$$

Since the left and right hand limits are equal, we can conclude that the two-sided limit does exist and equals the common value 0.

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$$\lim_{x \to 0} \frac{|x|}{x}$$

Note that the limiting value of x, 0, is not in the domain of the function $\frac{|x|}{x}$.
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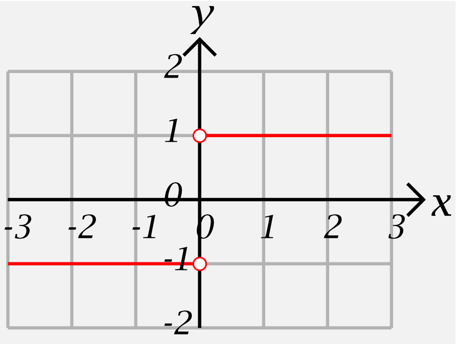

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$$\lim_{x \to 0^+} \frac{|x|}{x} =$$

Since the left and right hand limits are equal, we can conclude that the two-sided limit does exist and equals the common value 0.

Example 7:

$$\lim_{x\to 0} \frac{|x|}{x}$$

Note that the limiting value of x, 0, is not in the domain of the function $\frac{|x|}{x}$.

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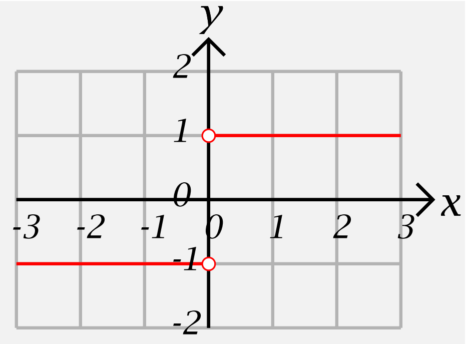

$$\lim_{x\to 0^-} \frac{|x|}{x} = \lim_{x\to 0^-} \frac{-x}{x} = \lim_{x\to 0^-} -1 = -1$$

$$\lim_{x\to 0^+} \frac{|x|}{x} = \lim_{x\to 0^+} \frac{x}{x} =$$

Since the left and right hand limits are equal, we can conclude that the two-sided limit does exist and equals the common value 0.

Example 7:

$\lim_{x\to 0} \frac{|x|}{x}$

Note that the limiting value of x, 0, is not in the domain of the function $\frac{|x|}{x}$.
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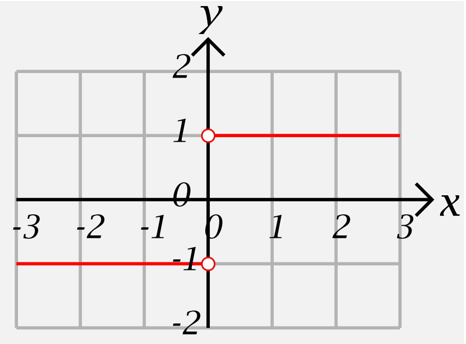

$$\lim_{x\to 0^-} \frac{|x|}{x} = \lim_{x\to 0^-} \frac{-x}{x} = \lim_{x\to 0^-} -1 = -1$$

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Since the left and right hand limits are equal, we can conclude that the two-sided limit does exist and equals the common value 0.

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$$\lim_{x\to 0}\frac{|x|}{x}$$

Note that the limiting value of x, 0, is not in the domain of the function $\frac{|x|}{x}$.

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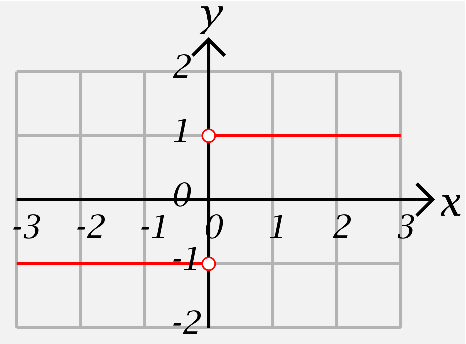

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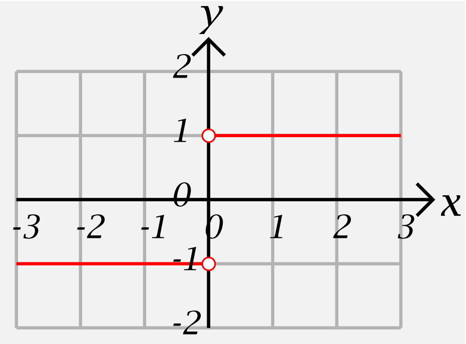

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Since the left and right hand limits are unequal, we can conclude that the two-sided limit does not exist.

Limits of Rational Functions

If $f(x) = \dfrac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials, then

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Rational Functions at $\pm\infty$

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Example 9: The **Floor and Ceiling Functions** .

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We define $\lfloor x \rfloor$, the floor of x, to be the largest integer that is less than or equal to x, and $\lceil x \rceil$, the ceiling of x, to be the least integer that is greater than or equal to x. In other words, the floor function rounds x down to the nearest integer and the ceiling function rounds x up to the nearest integer. They are both examples of step functions. They both have two-sided limits at non-integer values:

If a is not an integer, then $\lim_{x\to a}\lfloor x \rfloor = \lfloor a \rfloor$, and $\lim_{x\to a}\lceil x \rceil = \lceil a \rceil$, but if a is an integer, things are different:

(2) if the degree of $p(x)$ is greater than the degree of $q(x)$, $\lim_{x\to\pm\infty} f(x) = \pm\infty$,

(3) if the degree of $p(x)$ equals the degree of $q(x)$, $\lim_{x\to\pm\infty} f(x) = \dfrac{a_0}{b_0}$,
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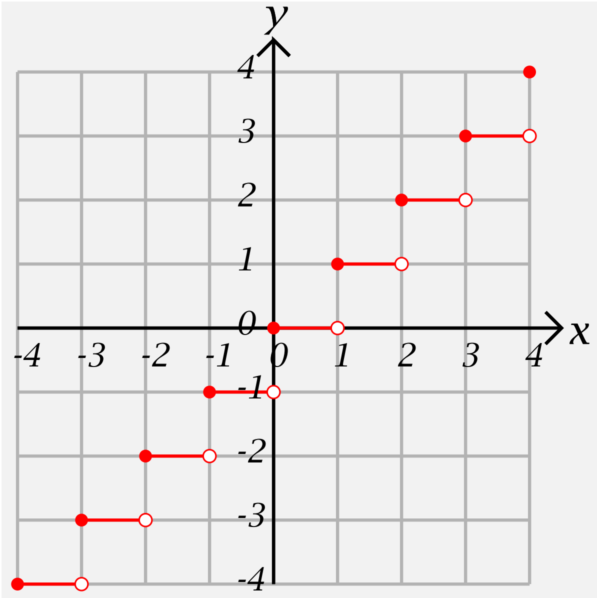
The Floor Function:



The Ceiling Function: