Exercises for Limit Laws

Find the indicated limits:

(1) $\lim_{x \to 1} \left[x^5 - 3x^3 + 1\right](x^2 - 2)$ Solution

(2) $\lim_{x \to 16} \frac{\sqrt{x}}{x + 16}$ Solution

(3) $\lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16}$ Solution

(4) $\lim_{x \to 3} \frac{x - 3}{x^2 - 9}$ Solution

(5) $\lim_{x \to 2} \frac{x^2 - 4}{x - 4} =$ Solution

(6) $\lim_{x \to 1^-} \left\lfloor \frac{x-1}{2} \right\rfloor$ Solution

---

(7) $\lim_{x \to 1^-} \frac{|x-1|}{x-1}$ Solution

---

(8) $\lim_{x \to 1^+} \frac{|x-1|}{x-1}$ Solution

---

(9) $\lim_{x \to \infty} \frac{x^2+1+1}{1-x}$ Solution

---

(10) $\lim_{x \to -\infty} \frac{x-1}{x^2-3x-1}$ Solution

Solutions

(1)

$$\lim_{x \to 1} \left[x^5 - 3x^3 + 1 \right] (x^2 - 2) =$$

Back to Questions

Solutions

(1)

$$\lim_{x\to 1}\left[x^5 - 3x^3 + 1\right](x^2 - 2) = \left[(1)^5 - 3(1)^3 + 1\right]((1)^2 - 2) =$$

Back to Questions

Solutions

(1)

$\lim_{x \to 1} \left[x^5 - 3x^3 + 1\right](x^2 - 2) = [(1)^5 - 3(1)^3 + 1]((1)^2 - 2) =$

Back to Questions

$[1 - 3 + 1](1 - 2)$

Solutions

(1)

$$\lim_{x \to 1}\left[x^5 - 3x^3 + 1\right](x^2 - 2) = [(1)^5 - 3(1)^3 + 1]((1)^2 - 2) =$$

Back to Questions

$$[1 - 3 + 1](1 - 2) = [-1](-1) =$$

Solutions

(1)

$$\lim_{x \to 1} \left[x^5 - 3x^3 + 1 \right] (x^2 - 2) = [(1)^5 - 3(1)^3 + 1] ((1)^2 - 2) =$$

Back to Questions

$$[1 - 3 + 1] (1 - 2) = [-1] (-1) = 1$$

(2)

$$\lim_{x \to 16} \frac{\sqrt{x}}{x + 16} =$$

Back to Questions

(2)

$$\lim_{x \to 16} \frac{\sqrt{x}}{x + 16} = \frac{\sqrt{16}}{16 + 16} =$$

Back to Questions

(2)

$$\lim_{x\to 16} \frac{\sqrt{x}}{x+16} = \frac{\sqrt{16}}{16+16} = \frac{4}{32} =$$

Back to Questions

(2)

$$\lim_{x \to 16} \frac{\sqrt{x}}{x + 16} = \frac{\sqrt{16}}{16 + 16} = \frac{4}{32} = \frac{1}{8}$$

Back to Questions

(3)

$$\lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16} =$$

Back to Questions

(3)

$$\lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16} =$$

$$\lim_{x \to 16} \left(\frac{\sqrt{x} - 4}{x - 16} \right) \left(\frac{\sqrt{x} + 4}{\sqrt{x} + 4} \right) =$$

Back to Questions

(3)

$$\lim_{x\to 16} \frac{\sqrt{x}-4}{x-16} =$$

$$\lim_{x\to 16} \left(\frac{\sqrt{x}-4}{x-16}\right)\left(\frac{\sqrt{x}+4}{\sqrt{x}+4}\right) = \lim_{x\to 16} \frac{(\sqrt{x})^2-4^2}{(x-16)(\sqrt{x}+4)} =$$

Back to Questions

(3)

$$\lim_{x\to 16} \frac{\sqrt{x}-4}{x-16} =$$

$$\lim_{x\to 16} \left(\frac{\sqrt{x}-4}{x-16}\right)\left(\frac{\sqrt{x}+4}{\sqrt{x}+4}\right) = \lim_{x\to 16} \frac{(\sqrt{x})^2-4^2}{(x-16)(\sqrt{x}+4)} =$$

Back to Questions

$$\lim_{x\to 16} \frac{x-16}{(x-16)(\sqrt{x}+4)} =$$

(3)

$$\lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16} =$$

$$\lim_{x \to 16} \left(\frac{\sqrt{x} - 4}{x - 16} \right) \left(\frac{\sqrt{x} + 4}{\sqrt{x} + 4} \right) = \lim_{x \to 16} \frac{(\sqrt{x})^2 - 4^2}{(x - 16)(\sqrt{x} + 4)} =$$

Back to Questions

$$\lim_{x \to 16} \frac{x - 16}{(x - 16)(\sqrt{x} + 4)} = \lim_{x \to 16} \frac{1}{\sqrt{x} + 4} =$$

(3)

$$\lim_{x\to 16} \frac{\sqrt{x}-4}{x-16} =$$

$$\lim_{x\to 16} \left(\frac{\sqrt{x}-4}{x-16}\right)\left(\frac{\sqrt{x}+4}{\sqrt{x}+4}\right) = \lim_{x\to 16} \frac{(\sqrt{x})^2-4^2}{(x-16)(\sqrt{x}+4)} =$$

Back to Questions

$$\lim_{x\to 16} \frac{x-16}{(x-16)(\sqrt{x}+4)} = \lim_{x\to 16} \frac{1}{\sqrt{x}+4} = \frac{1}{\lim\limits_{x\to 16}(\sqrt{x}+4)} =$$

(3)

$$\lim_{x\to 16} \frac{\sqrt{x}-4}{x-16} =$$

$$\lim_{x\to 16} \left(\frac{\sqrt{x}-4}{x-16}\right)\left(\frac{\sqrt{x}+4}{\sqrt{x}+4}\right) = \lim_{x\to 16} \frac{(\sqrt{x})^2-4^2}{(x-16)(\sqrt{x}+4)} =$$

Back to Questions

$$\lim_{x\to 16} \frac{x-16}{(x-16)(\sqrt{x}+4)} = \lim_{x\to 16} \frac{1}{\sqrt{x}+4} = \frac{1}{\lim\limits_{x\to 16}(\sqrt{x}+4)} = \frac{1}{\sqrt{16}+4} =$$

(3)

$$\lim_{x\to 16} \frac{\sqrt{x}-4}{x-16} =$$

$$\lim_{x\to 16} \left(\frac{\sqrt{x}-4}{x-16}\right)\left(\frac{\sqrt{x}+4}{\sqrt{x}+4}\right) = \lim_{x\to 16} \frac{(\sqrt{x})^2-4^2}{(x-16)(\sqrt{x}+4)} =$$

Back to Questions

$$\lim_{x\to 16} \frac{x-16}{(x-16)(\sqrt{x}+4)} = \lim_{x\to 16} \frac{1}{\sqrt{x}+4} = \frac{1}{\lim_{x\to 16}(\sqrt{x}+4)} = \frac{1}{\sqrt{16}+4} = \frac{1}{4+4} =$$

(3)

$$\lim_{x\to 16}\frac{\sqrt{x}-4}{x-16}=$$

$$\lim_{x\to 16}\left(\frac{\sqrt{x}-4}{x-16}\right)\left(\frac{\sqrt{x}+4}{\sqrt{x}+4}\right)=\lim_{x\to 16}\frac{(\sqrt{x})^2-4^2}{(x-16)(\sqrt{x}+4)}=$$

Back to Questions

$$\lim_{x\to 16}\frac{x-16}{(x-16)(\sqrt{x}+4)}=\lim_{x\to 16}\frac{1}{\sqrt{x}+4}=\frac{1}{\lim\limits_{x\to 16}(\sqrt{x}+4)}=\frac{1}{\sqrt{16}+4}=\frac{1}{4+4}=\frac{1}{8}$$

(4) $\lim_{x \to 3} \dfrac{x-3}{x^2-9} =$

Back to Questions

(4) $$\lim_{x\to 3}\frac{x-3}{x^2-9} = \lim_{x\to 3}\frac{x-3}{(x-3)(x+3)} =$$

Back to Questions

(4) $$\lim_{x\to 3} \frac{x-3}{x^2-9} = \lim_{x\to 3} \frac{x-3}{(x-3)(x+3)} = \lim_{x\to 3} \frac{1}{x+3} =$$

Back to Questions

(4) $$\lim_{x\to 3}\frac{x-3}{x^2-9}=\lim_{x\to 3}\frac{x-3}{(x-3)(x+3)}=\lim_{x\to 3}\frac{1}{x+3}=\frac{1}{3+3}=$$

Back to Questions

(4) $$\lim_{x\to 3}\frac{x-3}{x^2-9} = \lim_{x\to 3}\frac{x-3}{(x-3)(x+3)} = \lim_{x\to 3}\frac{1}{x+3} = \frac{1}{3+3} = \frac{1}{6}$$

Back to Questions

(5)

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} =$$

Back to Questions

(5)

$$\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(x+2)(x-2)}{x-2} =$$

Back to Questions

(5)

$$\lim_{x\to 2}\frac{x^2-4}{x-2}=\lim_{x\to 2}\frac{(x+2)(x-2)}{x-2}=\lim_{x\to 2}(x+2)=$$

Back to Questions

(5)

$$\lim_{x\to 2}\frac{x^2-4}{x-2}=\lim_{x\to 2}\frac{(x+2)(x-2)}{x-2}=\lim_{x\to 2}(x+2)=2+2=$$

Back to Questions

(5)

$$\lim_{x\to 2}\frac{x^2-4}{x-2} = \lim_{x\to 2}\frac{(x+2)(x-2)}{x-2} = \lim_{x\to 2}(x+2) = 2+2 = 4$$

Back to Questions

(6)

$$\lim_{x \to 1^-} \left\lfloor \frac{x-1}{2} \right\rfloor$$

Back to Questions

(6)

$$\lim_{x \to 1^-} \left\lfloor \frac{x-1}{2} \right\rfloor$$

Since x approaches 1 from the left,

Back to Questions

(6)

$$\lim_{x \to 1^-} \left\lfloor \frac{x-1}{2} \right\rfloor$$

Since x approaches 1 from the left, $x - 1$ will be negative,

Back to Questions

(6)

$$\lim_{x \to 1^-} \left\lfloor \frac{x-1}{2} \right\rfloor$$

Since x approaches 1 from the left, $x - 1$ will be negative, so $\left\lfloor \frac{x-1}{2} \right\rfloor = -1$ when x is in $(-1, 1)$.

Back to Questions

(6)

$$\lim_{x\to 1^-} \left\lfloor \frac{x-1}{2} \right\rfloor$$

Since x approaches 1 from the left, $x - 1$ will be negative, so $\left\lfloor \frac{x-1}{2} \right\rfloor = -1$ when x is in $(-1, 1)$. Thus

Back to Questions

(6)

$$\lim_{x\to 1^-} \left\lfloor \frac{x-1}{2} \right\rfloor$$

Since x approaches 1 from the left, $x-1$ will be negative, so $\left\lfloor \frac{x-1}{2} \right\rfloor = -1$ when x is in $(-1, 1)$. Thus

Back to Questions

$$\lim_{x\to 1^-} \left\lfloor \frac{x-1}{2} \right\rfloor =$$

(6)

$$\lim_{x\to 1^-} \left\lfloor \frac{x-1}{2} \right\rfloor$$

Since x approaches 1 from the left, $x - 1$ will be negative, so $\left\lfloor \frac{x-1}{2} \right\rfloor = -1$ when x is in $(-1, 1)$. Thus

Back to Questions

$$\lim_{x\to 1^-} \left\lfloor \frac{x-1}{2} \right\rfloor = \lim_{x\to 1^-} (-1) =$$

(6)

$$\lim_{x \to 1^-} \left\lfloor \frac{x-1}{2} \right\rfloor$$

Since x approaches 1 from the left, $x - 1$ will be negative, so $\left\lfloor \frac{x-1}{2} \right\rfloor = -1$ when x is in $(-1, 1)$. Thus

Back to Questions

$$\lim_{x \to 1^-} \left\lfloor \frac{x-1}{2} \right\rfloor = \lim_{x \to 1^-} (-1) = -1$$

(7) $\lim_{x \to 1^-} \frac{|x-1|}{x-1} =$

Back to Questions

(7) $\lim_{x \to 1^-} \frac{|x-1|}{x-1} = \lim_{x \to 1^-} \frac{1-x}{x-1} =$

Back to Questions

(7) $\lim_{x \to 1^-} \frac{|x-1|}{x-1} = \lim_{x \to 1^-} \frac{1-x}{x-1} = \lim_{x \to 1^-} -1 =$

Back to Questions

(7) $\lim_{x\to 1^-} \frac{|x-1|}{x-1} = \lim_{x\to 1^-} \frac{1-x}{x-1} = \lim_{x\to 1^-} -1 = -1$

Back to Questions

(8) $\lim_{x \to 1^+} \frac{|x-1|}{x-1} =$

Back to Questions

(8) $\lim_{x \to 1^+} \frac{|x-1|}{x-1} = \lim_{x \to 1^+} \frac{x-1}{x-1} =$

Back to Questions

(8) $$\lim_{x\to 1^+} \frac{|x-1|}{x-1} = \lim_{x\to 1^+} \frac{x-1}{x-1} = \lim_{x\to 1^+} 1 =$$

Back to Questions

(8) $$\lim_{x \to 1^+} \frac{|x-1|}{x-1} = \lim_{x \to 1^+} \frac{x-1}{x-1} = \lim_{x \to 1^+} 1 = 1$$

Back to Questions

(9) $\lim_{x \to \infty} \frac{x^2 + 1 + 1}{1 - x} =$

Back to Questions

(9) $\lim_{x\to\infty} \dfrac{x^2+1+1}{1-x} = -\infty$

Back to Questions

(10)

$$\lim_{x \to -\infty} \frac{x - 1}{x^2 - 3x - 1} =$$

Back to Questions

(10)

$$\lim_{x \to -\infty} \frac{x - 1}{x^2 - 3x - 1} = 0$$

Back to Questions