Exercises for Continuity

(1) Let $f(x) = \begin{cases} x^2 - 1 & \text{if } x \le 0 \\ 2x - c & \text{if } x > 0 \end{cases}$

Find the value of c that will make f continuous at 0.

Sketch the graph of f using this value of c. Solution

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(2) Let $f(x) = \begin{cases} x^2 - 2 & \text{if } x \le 0 \\ mx + b & \text{if } 0 < x < 1 \\ x^2 + 1 & \text{if } 1 \le x \end{cases}$

Find the values of m and b that will make f continuous at 0 and 1. Sketch the graph of f using these values of m and b. Solution

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(3) Let $f(x) = x - \left\lfloor \frac{x}{2} \right\rfloor$. Find the values where f is discontinuous. Sketch the graph of f. Solution

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(4) Let $f(x) = x^2 - \lfloor x^2 \rfloor$. Find the values where f is discontinuous. Sketch the graph of f. Solution

Solutions

(1)

$\lim_{x \to 0^-} f(x) =$

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Solutions

(1)

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x^2 - 1 =$$

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$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x^2 - 1 = 0^2 - 1 =$$

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Solutions

(1)

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x^2 - 1 = 0^2 - 1 = -1$, and $\lim_{x \to 0^+} f(x) =$

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Solutions

(1)

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x^2 - 1 = 0^2 - 1 = -1$, and $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} 2x - c =$

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Solutions

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$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x^2 - 1 = 0^2 - 1 = -1$, and $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 2x - c = 2(0) - c = -c$,

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Solutions

(1)

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x^2 - 1 = 0^2 - 1 = -1$, and $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 2x - c = 2(0) - c = -c$, so we must have

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$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x^2 - 1 = 0^2 - 1 = -1$, and $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 2x - c = 2(0) - c = -c$, so we must have $c = 1$.

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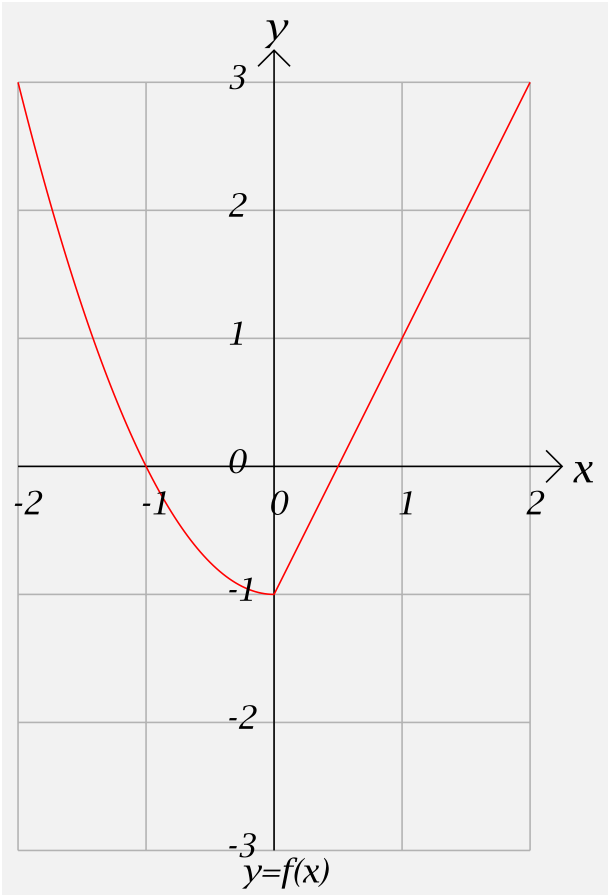

(2)

$$\lim_{x \to 0^-} f(x) =$$

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(2)

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x^2 - 2 =$$

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$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x^2 - 2 = 0^2 - 2 = -2$, and $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} mx + b = m(0) + b = b$, so we must have

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$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x^2 - 2 = 0^2 - 2 = -2$, and $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} mx + b = m(0) + b = b$, so we must have $b = -2$.

Also, $\lim_{x\to 1^-} f(x) =$

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Also, $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} mx + b = m(1) + b =$

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Also, $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} mx + b = m(1) + b = m + b = m - 2$, and $\lim_{x\to 1^+} f(x) =$

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$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x^2 - 2 = 0^2 - 2 = -2$, and $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} mx + b = m(0) + b = b$, so we must have $b = -2$.

Also, $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} mx + b = m(1) + b = m + b = m - 2$, and $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x^2 + 1 =$

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$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x^2 - 2 = 0^2 - 2 = -2$, and $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} mx + b = m(0) + b = b$, so we must have $b = -2$.

Also, $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} mx + b = m(1) + b = m + b = m - 2$, and $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x^2 + 1 = 1^2 + 1 =$

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$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x^2 - 2 = 0^2 - 2 = -2$, and $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} mx + b = m(0) + b = b$, so we must have $b = -2$.

Also, $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} mx + b = m(1) + b = m + b = m - 2$, and $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x^2 + 1 = 1^2 + 1 = 2$, so we must have

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$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x^2 - 2 = 0^2 - 2 = -2$, and $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} mx + b = m(0) + b = b$, so we must have $b = -2$.

Also, $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} mx + b = m(1) + b = m + b = m - 2$, and $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x^2 + 1 = 1^2 + 1 = 2$, so we must have $m - 2 = 2$ or

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$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x^2 - 2 = 0^2 - 2 = -2$, and $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} mx + b = m(0) + b = b$, so we must have $b = -2$.

Also, $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} mx + b = m(1) + b = m + b = m - 2$, and $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x^2 + 1 = 1^2 + 1 = 2$, so we must have $m - 2 = 2$ or $m = 4$.

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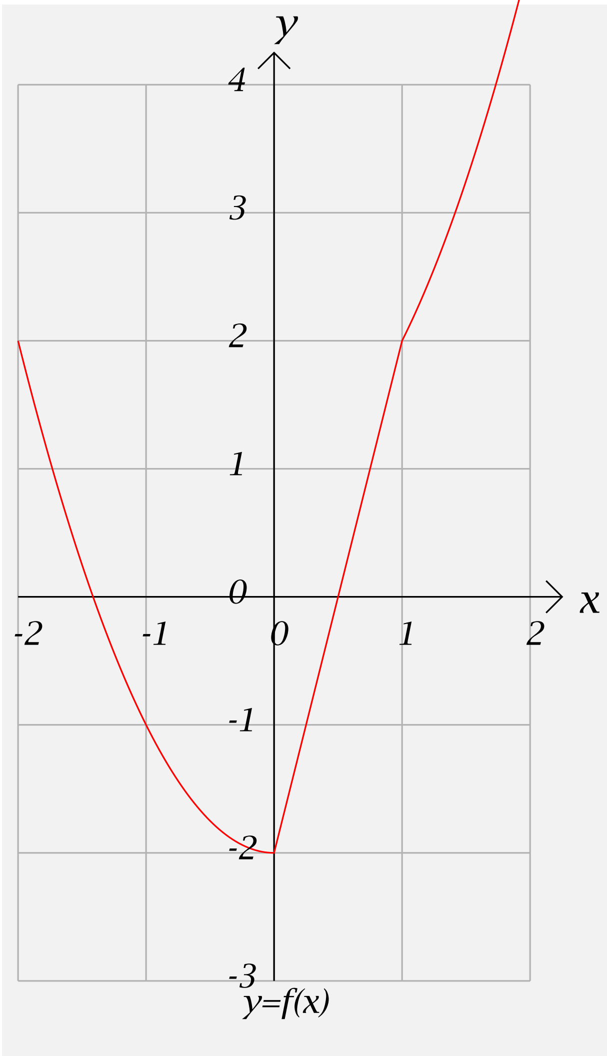

(3)

$\left\lfloor \frac{x}{2} \right\rfloor$ is discontinuous at all even integers.

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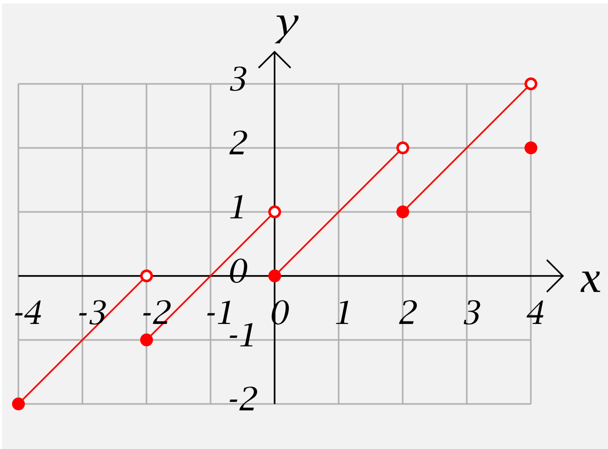

y=f(x)

(4)

$\lfloor x^2 \rfloor$ is discontinuous at all numbers whose square is an integer.

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