Applications of Logarithmic Functions:

Solutions to Extra Problems

Problem 1: A can of root beer is purchased from a dispenser at a temperature of 4°C and is left sitting in a room whose temperature is 24°C. In 15 minutes its temperature is 7°C. How much longer will it take to come to 12°C?

Solution: We use the equation $T(t) = T_\infty + (T_0 - T_\infty)e^{kt}$ with $T_0 = 4$ and $T_\infty = 24$:

$T(t) = 24 + (4 - 24)e^{kt} = 24 - 20e^{kt}$.

We then have $T(15) = 7 = 24 - 20e^{k(15)}$, so $e^{15k} = \dfrac{17}{20}$. Taking logs,

$15k = \ln\left(\dfrac{17}{20}\right) = \ln 17 - \ln 20$, and $k = \dfrac{\ln 17 - \ln 20}{15}$.

Therefore $T(t) = 24 - 20e^{\frac{\ln 17 - \ln 20}{15}t}$. We solve for the t which gives $T(t) = 12$:

$12 = 24 - 20e^{\frac{\ln 17 - \ln 20}{15}t}$, so $e^{\frac{\ln 17 - \ln 20}{15}t} = \dfrac{12 - 24}{-20} = 0.6$.

Taking logs, we get $\dfrac{\ln 17 - \ln 20}{15}t = \ln 0.6$, so

$t = 15\dfrac{\ln 0.6}{\ln 17 - \ln 20} \doteq 15\dfrac{-0.5108256}{2.8332133 - 2.9957323} \doteq 15\dfrac{-0.5108256}{-0.162519} \doteq 15(3.1431748) =$

47.147622 minutes

Problem 2: Dominic, the Vancouver coroner, is called at 7:00 A.M. to examine a corpse found in the harbour. Its temperature is found to be 22°C, and 5 minutes later is 21.5°C. The air and water temperature is 5°C. When was the time of death?

Solution: We use the equation $T(t) = T_\infty + (T_0 - T_\infty)e^{kt}$ with $T_0 = 22$ and $T_\infty = 5$:

$T(t) = 5 + (22 - 5)e^{kt} = 5 + 17e^{kt}$.

We then have $T(5) = 21.5 = 5 + 17e^{k(5)}$,

so $e^{5k} = \dfrac{21.5 - 5}{17} = \dfrac{16.5}{17}$,

and $5k = \ln\left(\dfrac{16.5}{17}\right) = \ln 16.5 - \ln 17$,

and therefore $k = \dfrac{\ln 16.5 - \ln 17}{5}$, so we get

$T(t) = 5 + 17e^{\frac{\ln 16.5 - \ln 17}{5}t}$.

Next we solve for the value of t for which $T(t) = 38$, the normal temperature of a normal human:

$38 = 5 + 17e^{\frac{\ln 16.5 - \ln 17}{5}t}$ if $\dfrac{33}{17} = e^{\frac{\ln 16.5 - \ln 17}{5}t}$. Taking logarithms:

$\ln\left(\dfrac{33}{17}\right) = \dfrac{\ln 16.5 - \ln 17}{5}t$, so

$t = 5\dfrac{\ln 33 - \ln 17}{\ln 16.5 - \ln 17} \doteq 5\dfrac{3.4965076 - 2.8332133}{2.8033604 - 2.8332133} \doteq 5\dfrac{0.6632943}{-0.0298529} \doteq 5(-22.218756) = -111.09378$ minutes, so the time of death was about **5:09 A.M.**

Problem 3: A pizza at the temperature of 20°C is placed in an oven whose temperature is 300°C. One minute later it is 30°C. It is considered to be cooked when its tempearure is 90°C. When will that be?

Solution: We use the equation $T(t) = T_\infty + (T_0 - T_\infty)e^{kt}$ with $T_0 = 20$ and $T_\infty = 300$:

$T(t) = 300 + (20 - 300)e^{kt} = 300 - 280e^{kt}$.

We then have $T(1) = 30 = 300 - 280e^{k(1)}$,

so $e^k = \dfrac{30 - 300}{-280} = \dfrac{270}{280} = \dfrac{27}{28}$ so we get $k = \ln 27 - \ln 28$

$T(t) = 300 - 280e^{(\ln 27 - \ln 28)t}$.

Next we solve for the value of t for which $T(t) = 90$:

$90 = 300 - 280e^{(\ln 27 - \ln 28)t}$ if $\dfrac{90 - 300}{-280} = e^{(\ln 27 - \ln 28)t}$, or

$\dfrac{21}{28} = \dfrac{3}{4} = e^{(\ln 27 - \ln 28)t}$,. Taking logarithms:

$\ln 3 - \ln 4 = (\ln 27 - \ln 28)t$, so

$t = \dfrac{\ln 3 - \ln 4}{\ln 27 - \ln 28} \doteq \dfrac{1.0986123 - 1.2862944}{3.2958369 - 3.3322045} \doteq \dfrac{-0.2876821}{-0.0363676} =$ 7.9103955 minutes.

Problem 4: The proportion of radioactive Carbon-14 in the atmosphere is maintained at a constant level by cosmic radiation. When living organisms die, they stop absorbing carbon, so the amount of radioactive carbon they contain decays exponentially from that time to zero. Carbon-14 has a half-life of 5730 years. A roof-beam in an old Viking structure in Iceland was found to have 90% of the proportion of Carbon-14 found in a living tree. When was the tree it came from cut down?

Solution: Using the equation $x(t) = x(0)e^{kt}$, we have $\frac{1}{2}x(0) = x(5730) = x(0)e^{k(5730)}$, so

$\frac{1}{2} = e^{5730k}$. Takin logarithms, we get $\ln\left(\frac{1}{2}\right) = -\ln 2 = 5730k$, so $k = -\frac{\ln 2}{5730}$, and thus $x(t) = x(0)e^{-\frac{\ln 2}{5730}t}$.

We solve for the value of t that gives $x(t) = 0.90x(0)$:

$0.90x(0) = x(0)e^{-\frac{\ln 2}{5730}t}$ or $0.90 = e^{-\frac{\ln 2}{5730}t}$. Taking logs, we get

$\ln 0.90 = -\frac{\ln 2}{5730}t$, so

$t = -5730\frac{\ln 0.90}{\ln 2} \doteq -5730\frac{-0.1053605}{0.6931471} = -5730(-0.1520031) =$ **870.97776 years.**

Problem 5: Radioactive waste is accidentally spilled into a lake in Northern Saskatchewan. The volume of water in the lake turns over once every 25 years. How long will it take for the concentration of radioactive waste in the lake to drop to 1% of its level right after the spill?

Solution: We use the equation $y(t) = y(0)e^{-\frac{t}{f}} = y(0)e^{-\frac{t}{25}}$ and solve for the value of t that gives $y(t) = 0.01y(0)$:

$0.01y(0) = y(0)e^{-\frac{t}{25}}$ or $0.01 = e^{-\frac{t}{25}}$. Taking logs, we get:

$\ln 0.01 = -\dfrac{t}{25}$, so $t = -25\ln 0.01 \doteq -25(-4.6051702) =$ **115.12926 years.**