Functions

Functions arise whenever one quantity depends on another.

1. The area A of a circle depends on its radius: $A = \pi r^2$
2. The human population P of the world depends on the time t. There is no formula for P as a function of t, but we can still say $P(2001) \doteq 6,000,000,000$
3. The cost C of mailing a first-class letter depends on the weight w of the letter. Although there is no simple formula connecting w and C, the post office has a rule for determining C when w is known.

A function f is a rule that assigns to each value x exactly one value y

The set in which the values of the **independent variable** x lie is called the **domain** of f and the set of values y of the **dependent** variable is called the **range**. We will write $f(x)$ for the value of y associated with x, and will often write $y = f(x)$.

The function's domain may be explicitly specified. If it is not then it is always assumed to be the largest set of numbers for which the given rule makes sense.

Functions can be thought of as **machines**, and can be depicted (inefficiently) with **arrow diagrams**.

The most common and efficient way of depicting a function is with its **graph**, the set of points

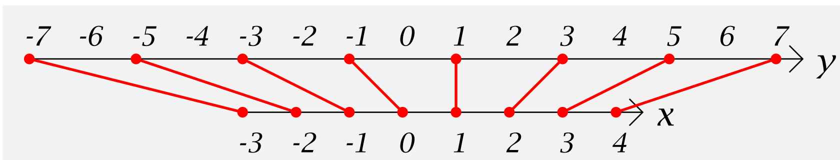
$$\{(x, y) : y = f(x) \text{ and } x \in \text{domain of } f\}$$

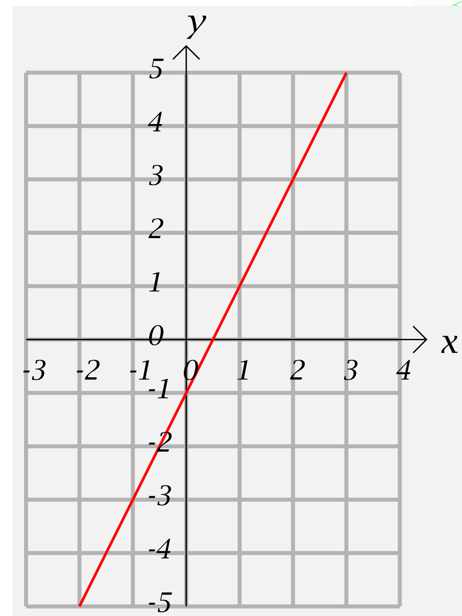
One important application of Calculus will be the development of techniques for sketching graphs of functions.

Example 1: Sketch the graph of the function given by the formula $f(x) = 2x - 1$. What are the domain and range? What is $f(-2)$? What is $f(3)$? Give a verbal rule for the function, and make a table of values.

Solution: Since $2x - 1$ is defined for all real numbers, the domain is $(-\infty, \infty)$. To determine the range is a bit more difficult, but there is a simple way of telling whether or not a number c is in the range: If the horizontal line $y = c$ crosses the graph, then c is a value of the function. Since all horizontal lines cross the graph of $y = 2x - 1$, the range consists of all real numbers, and is therefore $(-\infty, \infty)$. $f(-2) = 2(-2) - 1 = -4 - 1 = -5$, and $f(3) = 2(3) - 1 = 6 - 1 = 5$. A verbal rule is "Multiply by two and subtract one."

x	-3	-2	-1	0	1	2	3	4
$f(x)$	-7	-5	-3	-1	1	3	5	7

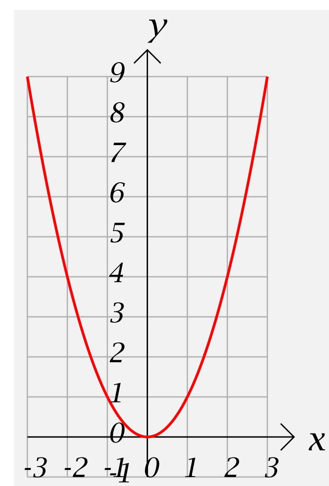


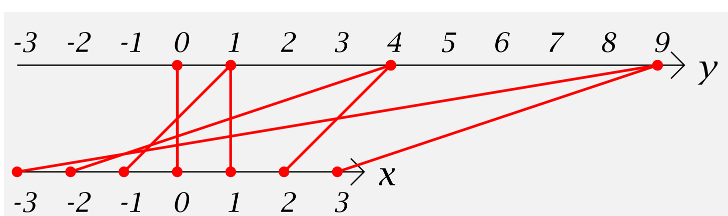


Example 2: Sketch the graph of the function given by the formula $f(x) = x^2$. What are the domain and range? What is $f(-2)$? What is $f(3)$? Give a verbal rule for the function, and make a table of values.

Solution:
Since x^2 is defined for all real numbers, the domain is $(-\infty, \infty)$. Since all horizontal lines lying at or above the x-axis cross the graph of $y = x^2$, the range consists of all non-negative real numbers, and is therefore the interval $[0, \infty)$. $f(-2) = (-2)(-2) = 4$, and $f(3) = 3(3) = 9$. A verbal rule is "Multiply x by itself."

x	-3	-2	-1	0	1	2	3
$f(x)$	9	4	1	0	1	4	9





The Four Ways to Represent a Function

- numerically—by giving a table of values
- visually—by giving a graph
- algebraically—by giving an explicit formula
- verbally—by giving a description in words