Properties of Functions

A lot of time can be saved in graphing and analyzing functions if we can observe some fundamental properties before beginning to calculate with them.

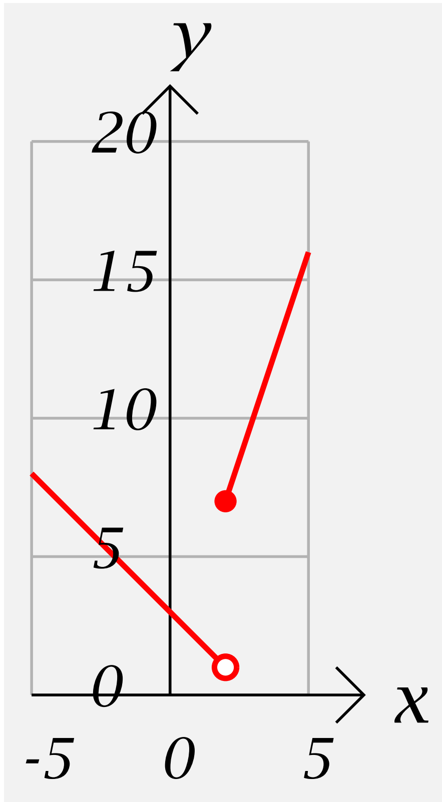
The Vertical Line Test

A curve in the xy-plane is the graph of a function of x if and only if no vertical line intersects the curve more than once.

Circles cannot be graphs of functions for this reason.

Piecewise Defined Functions

Functions can be, and often defined with "multiline formulas" which give different formulas for the value of the function in terms of the independent variable belonging to different parts (or pieces, hence the term "piece-wise"):



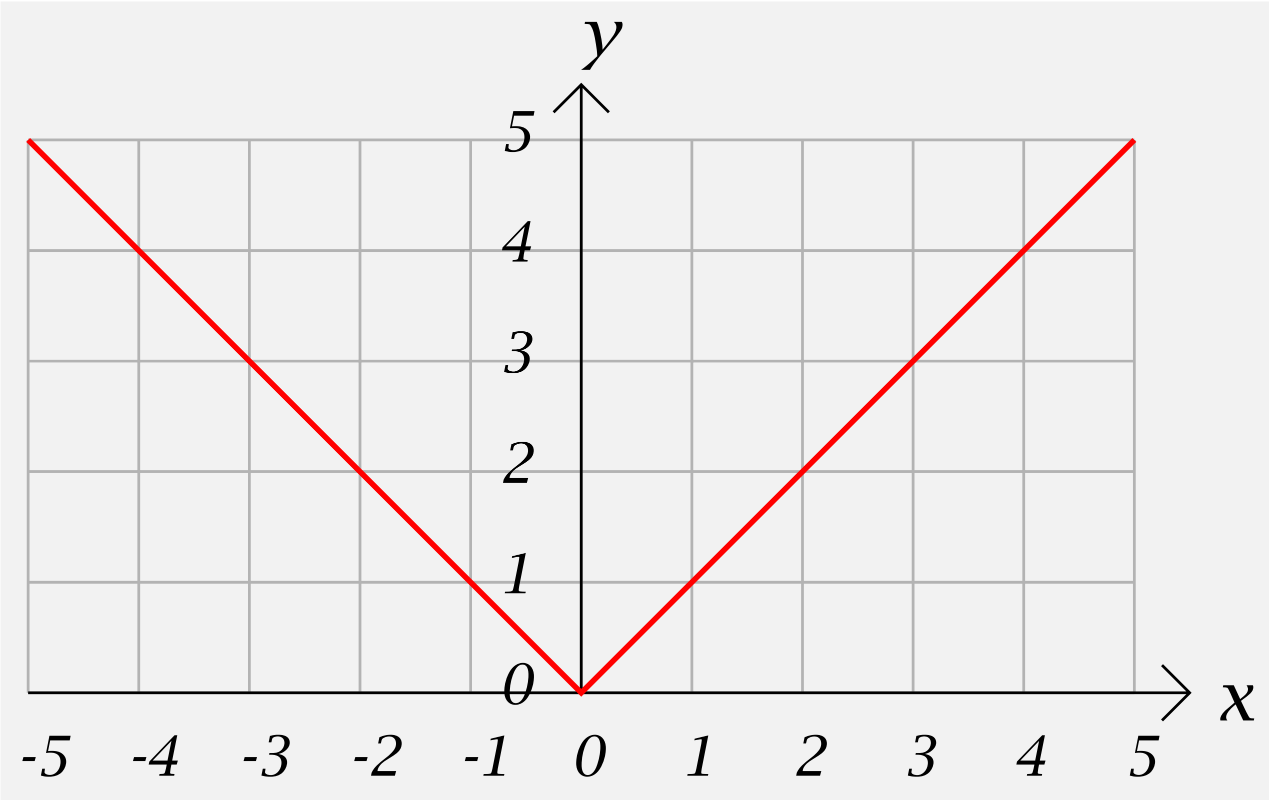
Example 1:

$$f(x) = \begin{cases} -x+3 & \text{if } x < 2 \\ 3x+1 & \text{if } x \geq 2 \end{cases}$$

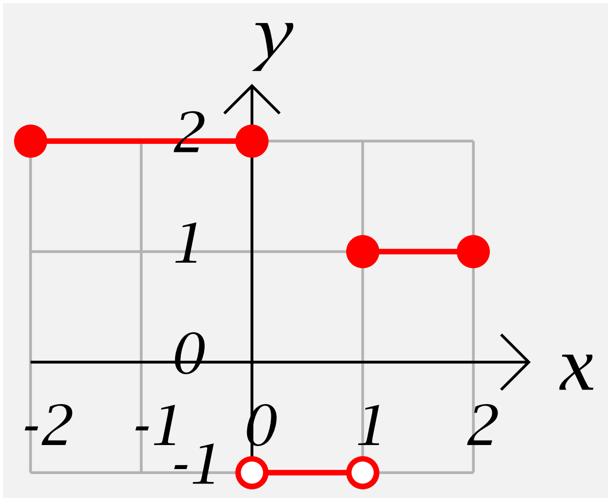
The domain is $(-\infty, \infty)$ and the range is $(1, \infty)$. Why?

We have already seen the absolute value function defined in this way:

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$



Functions defined by multiline formulas which give constant values on intervals are called step functions.



The Natural Domain

The function's domain may be explicitly specified. If it is not then it is always assumed to be the largest set of numbers for which the given rule makes sense. This set is called the natural domain of the function.

Symmetry

If a function f satisfies the equation $f(-x) = f(x)$ for all x in its domain, we say that f is even. Its graph is symmetric about the y-axis.

Examples are $f(x) = |x|$, since $f(-x) = |-x| = |x| = f(x)$,
$f(x) = x^2$, since $f(-x) = (-x)^2 = x^2 = f(x)$,
$f(x) = x^4 - x^2$, since
$f(-x) = (-x)^4 - (-x)^2 = x^4 - x^2 = f(x)$,
$f(x) = \sqrt{|x|}$ since $f(-x) = \sqrt{|-x|} = \sqrt{|x|} = f(x)$.
(Why not $f(x) = \sqrt{x}$?)

If a function f satisfies the equation $f(-x) = -f(x)$ for all x in its domain, we say that f is odd. Its graph is symmetric about the origin.

Examples are $f(x) = x$, since $f(-x) = -x = -f(x)$,
and $f(x) = x^3$ since $f(-x) = (-x)^3 = -x^3 = -f(x)$.

Students may see **animated examples of such functions** with a Java applet .