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Circles cannot be graphs of functions for this reason.

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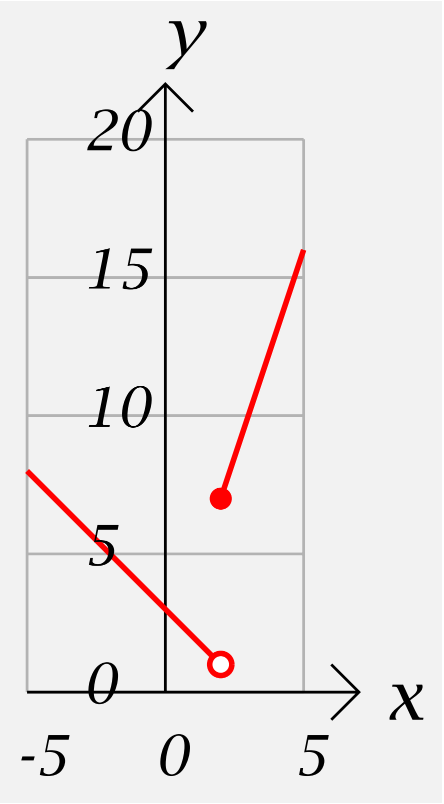

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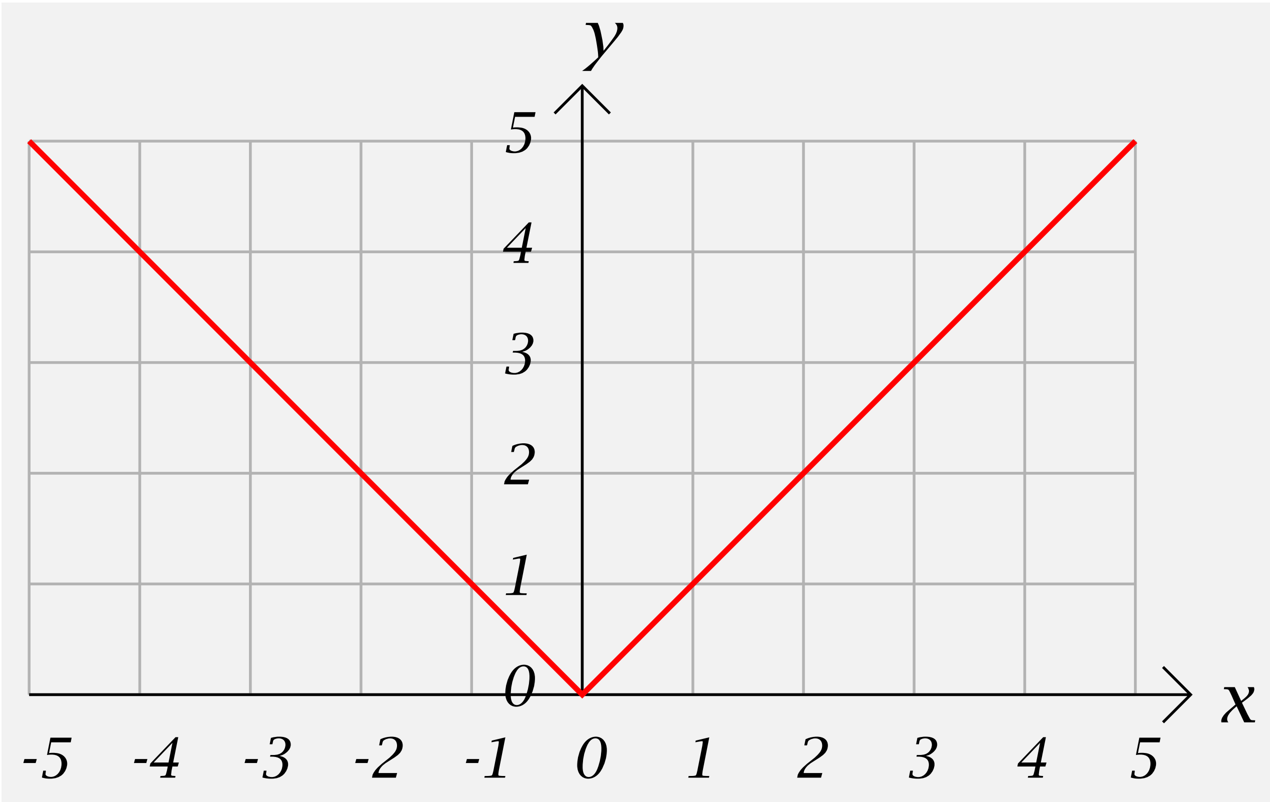

Properties of Functions-4

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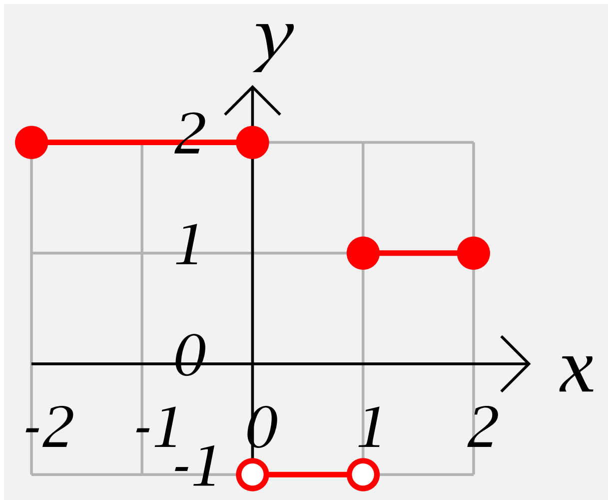

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If a function f satisfies the equation $f(-x) = -f(x)$ for all x in its domain, we say that f is odd. Its graph is symmetric about the origin.

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