

Properties of Functions

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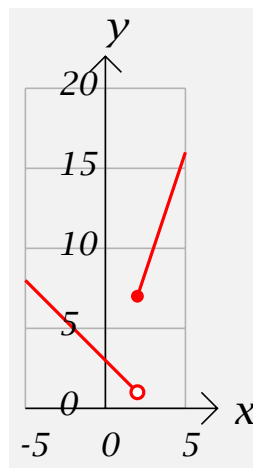
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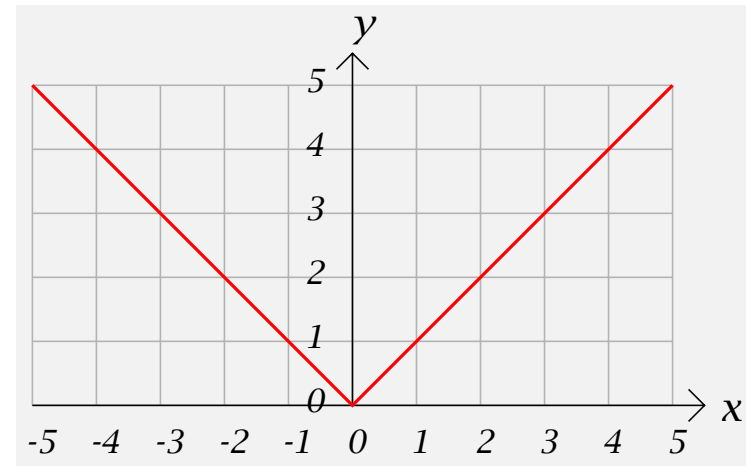
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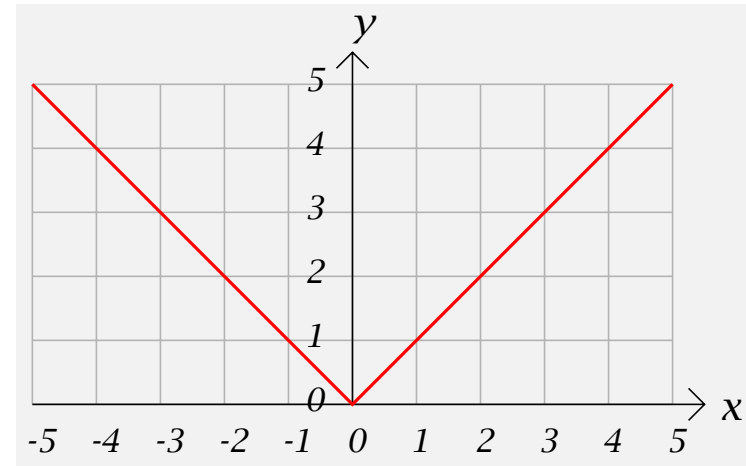
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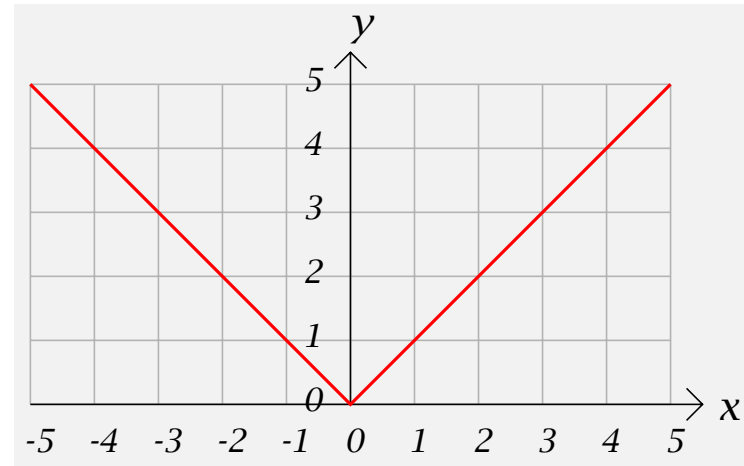
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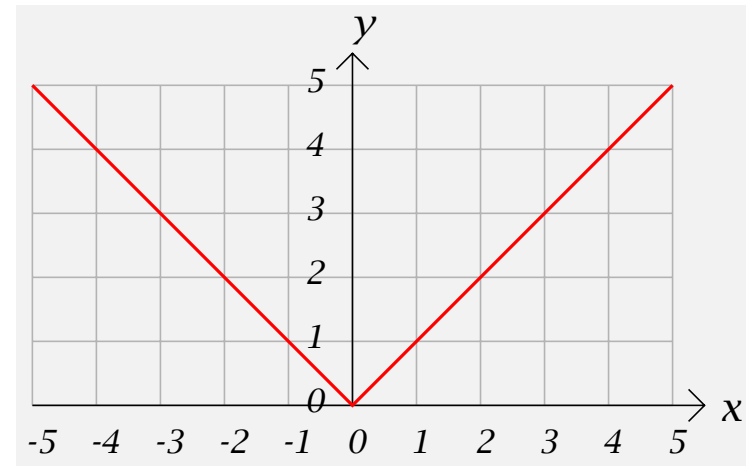
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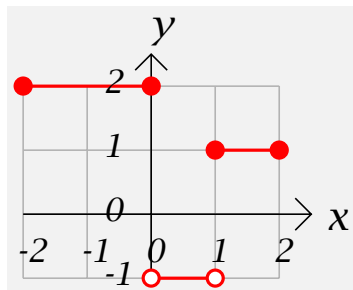
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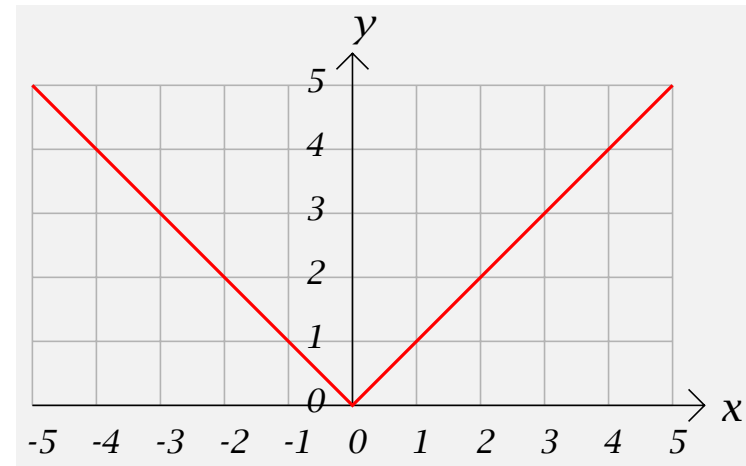


The Natural Domain

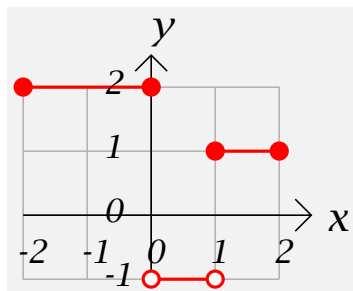
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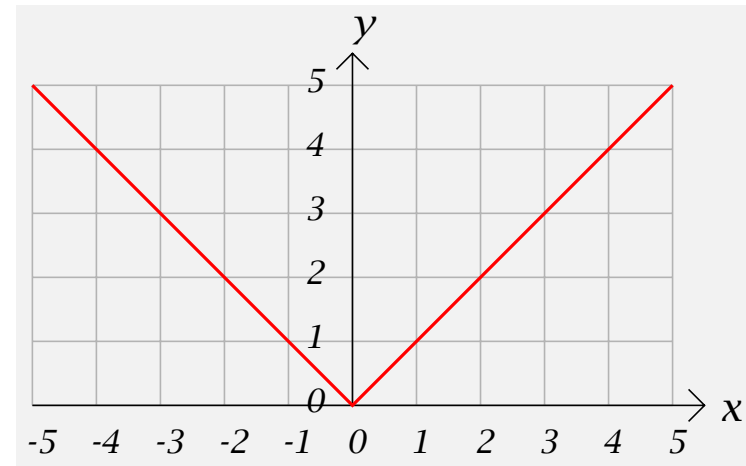


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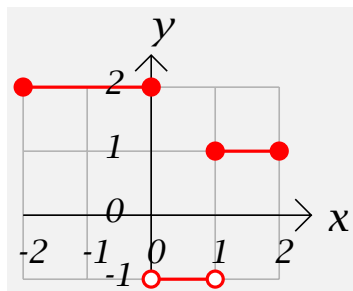
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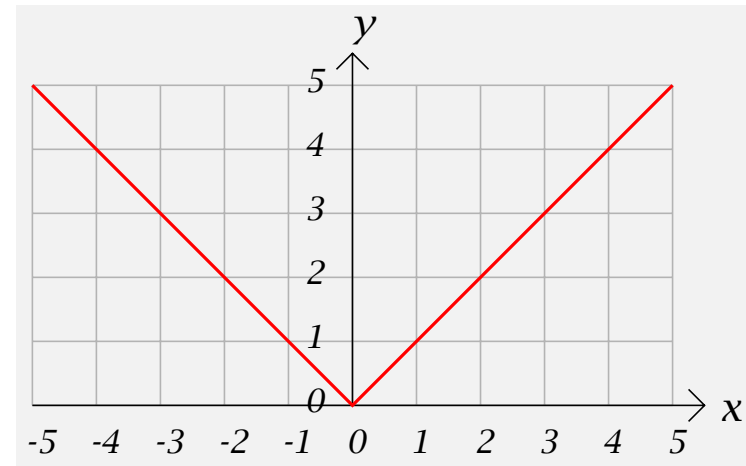


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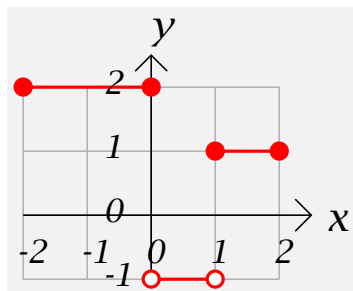
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The function's domain may be explicitly specified. If it is not then it is always assumed to be the largest set of numbers for which the given rule makes sense. This set is called the **natural domain** of the function.

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