UNIVERSITY OF SASKATCHEWAN
Department of Mathematics & Statistics
Mathematics 101.3 Final Examination

9 A.M.,December 9, 2000 Time:3 hours Instructor: *Doug MacLean*

CLOSED BOOK — CALCULATORS NOT PERMITTED

PART I

The possible answers to all questions in Part I are the digits in the **ANSWER SET:**

(A) 0	(B) 1	(C) 2	(D) 3	(E) 4	(F) 5	(G) 6	(H) 7	(I) 8	(J) 9

If $\frac{5}{11} - \frac{2}{5}$ is written in its simplest form as $\frac{a}{10b+c}$, where a, b, and c are digits, then

(1) $a =$ **(2)** $b =$ **(3)** $c =$

$2x^2 - 20x + 58$ is to be written in the form $a\left[(x-b)^2 + c^2\right]$ by completing squares. We must have:

(4) $a =$ **(5)** $b =$ **(6)** $c =$

$4x^2 + 40x + 96$ is to be written in the form $a(x+b)^2 - c$ by completing squares. We must have:

(7) $a =$ **(8)** $b =$ **(9)** $c =$

(10) If a is the slope of the line through the points $(-4, -12)$ and $(4, 12)$, then a is:

The roots of $2x^2 + x - 6 = 0$ in their simplest form are $-A$ and $\frac{B}{C}$. We must have:

(11) $A =$ **(12)** $B =$ **(13)** $C =$

The polynomial $p(x) = 10x^4 - 7x^3 - 9x^2 + 7x - 1$ can be factored in the form $(x-a)(x+b)(cx-1)(dx-1)$, where a, b, c, and d are digits, and $c < d$. Their values are:

(14) $a =$ **(15)** $b =$ **(16)** $c =$ **(17)** $d =$

…2

$\dfrac{3\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} - 2$ can be simplified to the expression $a - b\sqrt{c}$, where a, b, and c are digits. Their values are:

(18) $a =$ **(19)** $b =$ **(20)** $c =$

If we solve the inequality $\left|\dfrac{5-x}{4}\right| \geq 1$, the solution is an interval of the form $(-\infty, a] \cup [b, \infty)$.
The values of a and b are:

(21) $a =$ **(22)** $b =$

(23) The x-intercept of the line perpendicular to the line $y = -\dfrac{x}{14}$ which passes through the point $(9, 0)$ is:

Evaluate the limits:

(24) $\lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$ if $f(x) = x^3$ and $x = \dfrac{2}{\sqrt{3}}$

(25) $\lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$ if $f(x) = -\dfrac{7}{x+4}$ and $x = -5$

(26) $\lim\limits_{x\to -3} \dfrac{6x^2 + 30x - 84}{3x - 6}$

(27) $\lim\limits_{x\to -5} \dfrac{-15 - 8x - x^2}{x + 5}$

(28) The natural domain of the function $f(x) = \sqrt[8]{64x^2 - 1}$ is of the form $\left(-\infty, -\dfrac{1}{a}\right] \cup \left[\dfrac{1}{a}, \infty\right)$. What is the value of a =?

(29) The y−intercept of the line perpendicular to the line $\dfrac{x}{6} = y$ which passes through the point $(1, 2)$ is:

(30) Let $f(x) = \dfrac{x^4}{(x+1)^4}$. Find $64f'(1)$

(31) The minimum value of $f(x) = x^2 + 8x + 22$ is:

(32) Find the positive solution of the equation $2^{x^2-1} = 256$:

(33) The largest solution of $\ln(x+3) + \ln(x+7) = \ln(20x)$ is:

…3

If $\log_2 1024 = 10a + b$ then **(34)** $a =$ and **(35)** $b =$

If $\log_{25} 125 = \dfrac{c}{d}$ then **(36)** $c =$ and **(37)** $d =$

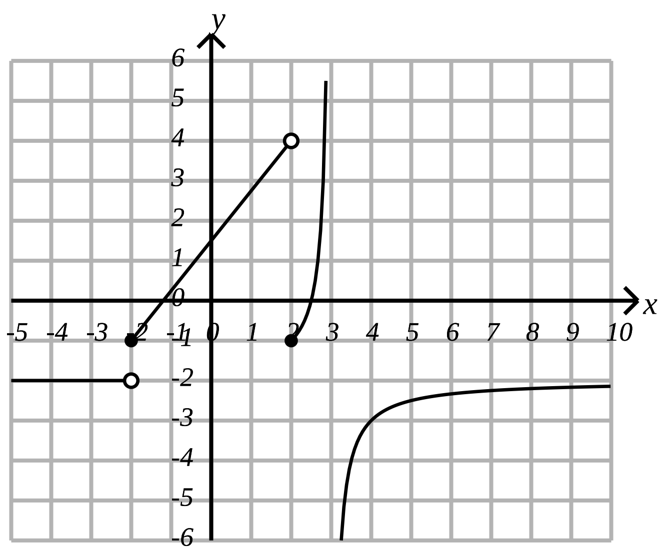
If $\log_{16} 128 = \dfrac{e}{f}$ then **(38)** $e =$ and **(39)** $f =$

PART II

The possible answers to all questions in Part II are in the **ANSWER SET:**

(A) $-\infty$ (B) -4 (C) -2 (D) -1 (E) 0 (F) 1 (G) 2 (H) 4 (I) 8 (J) ∞

(40) $\lim_{x\to -4^-} \dfrac{6x-1}{x+4} = ?$ **(41)** $\lim_{x\to -4^+} \dfrac{6x-1}{x+4} = ?$



Part of the graph of $y = f(x) = \begin{cases} -2 & \text{if } x < -2 \\ \dfrac{5x+6}{4} & \text{if } -2 \le x < 2 \\ \dfrac{1}{3-x} - 2 & \text{if } 2 \le x \text{ and } x \ne 3 \end{cases}$ is shown above.

Find:

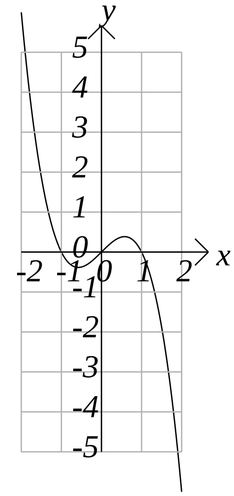
(42) $\lim_{x\to -\infty} f(x)$ **(43)** $\lim_{x\to -2^-} f(x)$ **(44)** $\lim_{x\to -2^+} f(x)$ **(45)** $\lim_{x\to 2^-} f(x)$

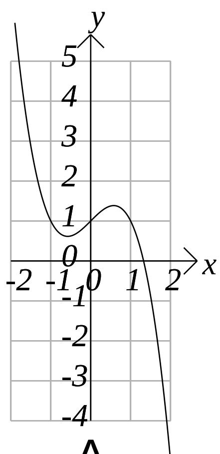
(46) $\lim_{x\to 2^+} f(x)$ **(47)** $\lim_{x\to 3^-} f(x)$ **(48)** $\lim_{x\to 3^+} f(x)$ **(49)** $\lim_{x\to \infty} f(x)$

…4

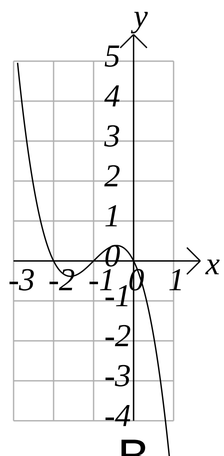
PART III

The graph of $y = f(x) = x - x^3$ is shown to the right. Parts of the graphs of
(50) $y = f(2x)$,
(51) $y = 2f(x)$,
(52) $y = f(x)/2$,
(53) $y = f(x+1)$,
(54) $y = f(x) - 1$,
(55) $y = f(x-1)$,
(56) $y = -f(x)$,
(57) $y = f(x/2)$, and
(58) $y = -f(x) + 1$, are shown below.
Match them.





A



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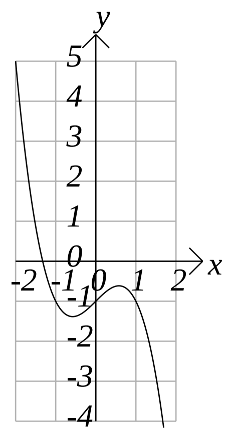

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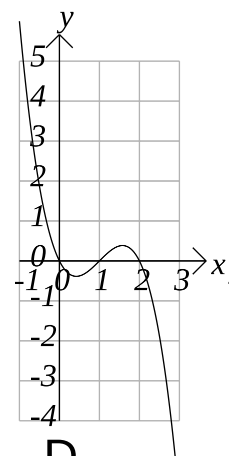
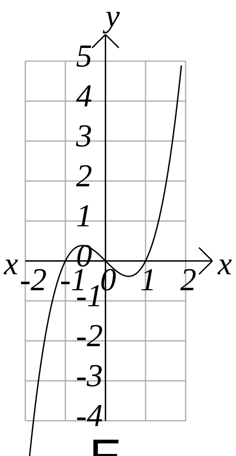
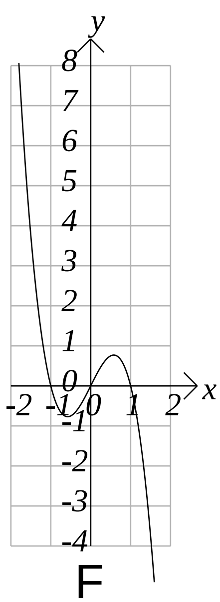


D



E



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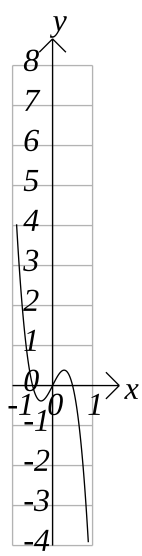
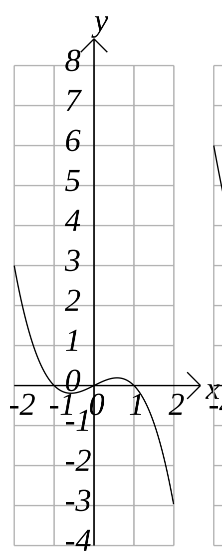
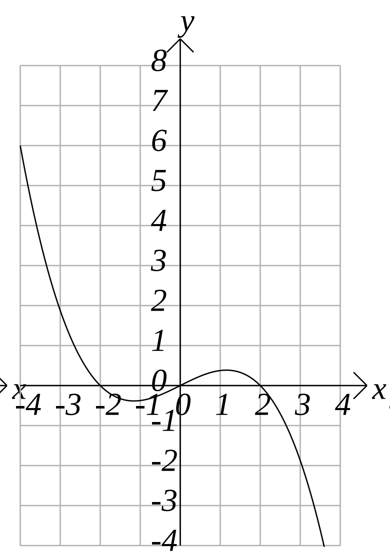


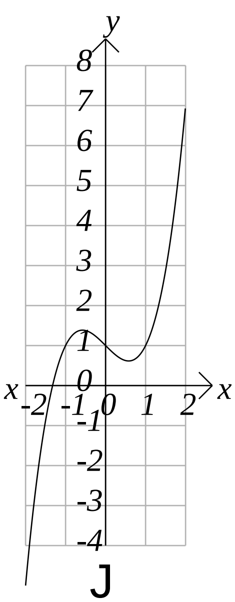
G



H



I



J

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PART IV

The graphs of the functions in questions 59 to 65 are shown below. Match them.

(59) $f(x) = \frac{1}{10}\left(2x^3 - 6x^2 + 3\right)$

(60) $f(x) = \frac{1}{10}\left(x^4 - 4x^2\right)$

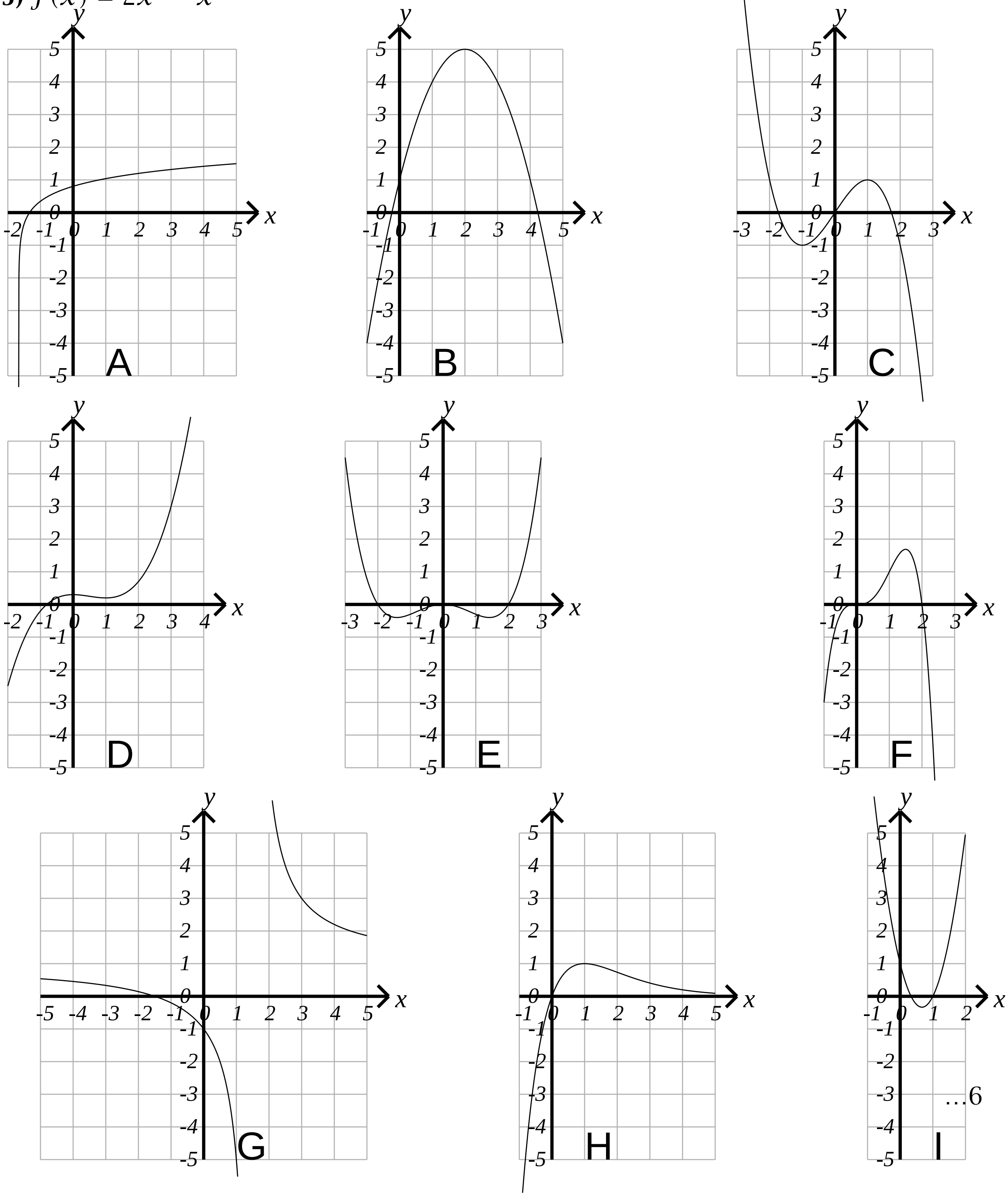
(61) $f(x) = \frac{1}{2}\left(3x - x^3\right)$

(62) $f(x) = \frac{2x+3}{2x-3}$

(63) $f(x) = 4x - x^2 + 1$

(64) $f(x) = 3x^2 - 4x + 1$

(65) $f(x) = 2x^3 - x^4$



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PART V

The graphs of the functions in questions 66 to 73 are shown below. Match them.

(66) $f(x) = 5e^{-\frac{1}{x^2}}$

(67) $f(x) = xe^{x}$

(68) $f(x) = 5x^2e^{-2x}$

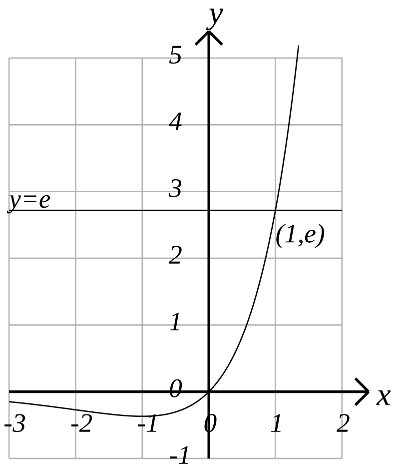
(69) $f(x) = 5x^2e^{2x}$

(70) $f(x) = 5xe^{-2x}$

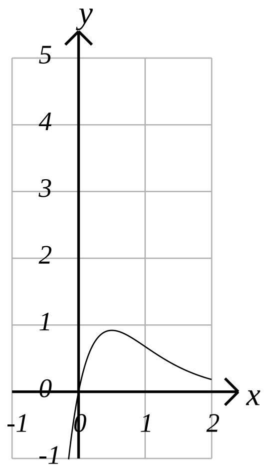
(71) $f(x) = 0.5 * xe^{x^2}$

(72) $f(x) = e^{x}$

(73) $f(x) = e^{-x}$



A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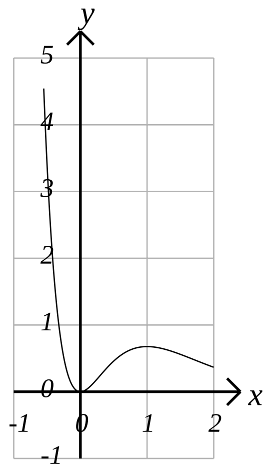
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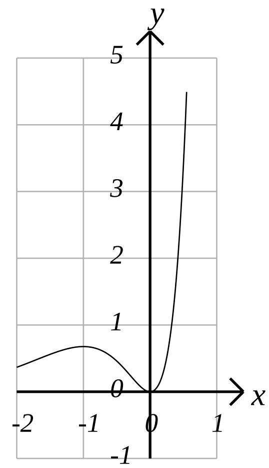
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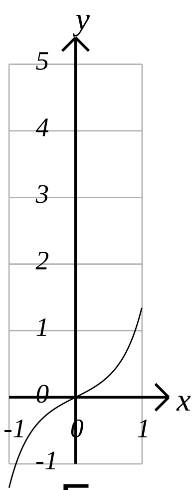
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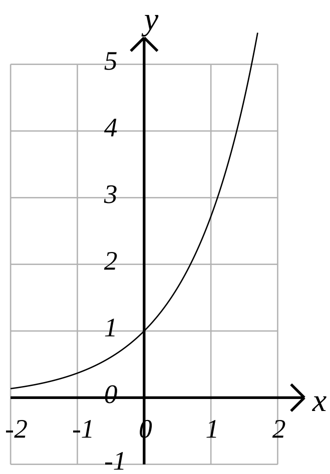
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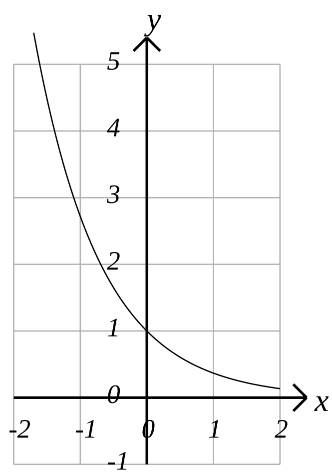
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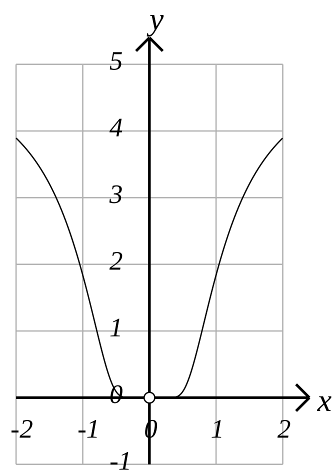
H

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PART VI

The graphs of the functions in questions 74 to 80 are shown below. Match them.

(74) $f(x) = \ln x^x$

(75) $f(x) = \ln x$

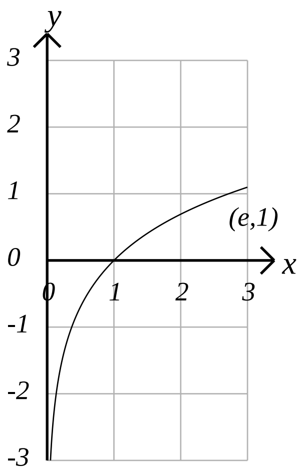
(76) $f(x) = \ln \sqrt{x}$

(77) $f(x) = x \ln x$

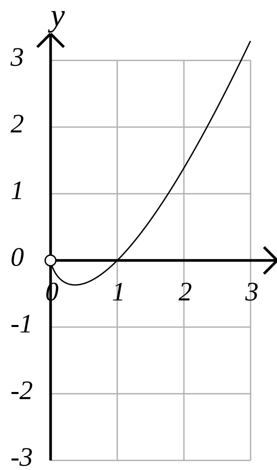
(78) $f(x) = \ln x^2$

(79) $f(x) = x^2 \ln x$

(80) $f(x) = \ln ex$



A



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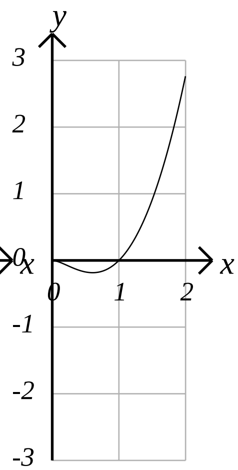

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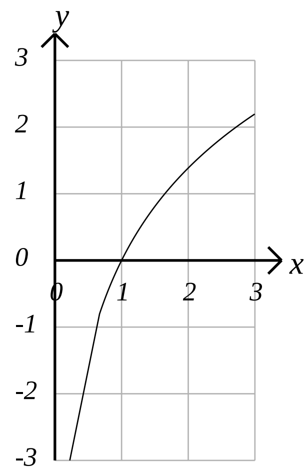

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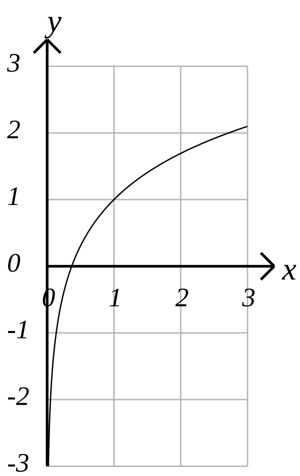

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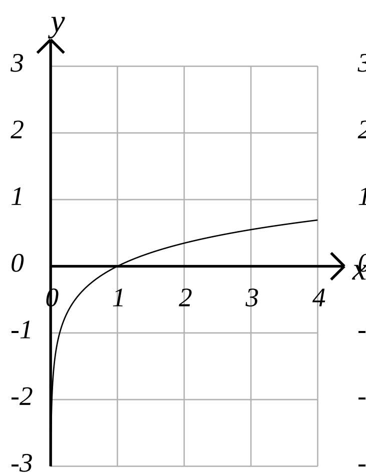

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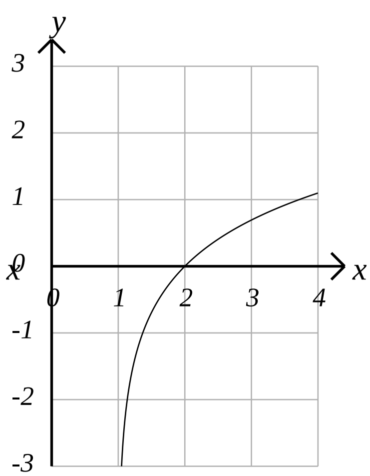

G

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PART VII

Each question in this section is worth 5 marks.

The possible answers to all questions in Part VII are the digits in the **ANSWER SET:**

(A) 0	(B) 1	(C) 2	(D) 3	(E) 4	(F) 5	(G) 6	(H) 7	(I) 8	(J) 9

(81) Find the absolute minimum value of the function $f(x) = 9 + (\ln 3x)^2$.

(82) Two positive numbers have their product equal to 16. What is the minimum possible value of their sum?

(83) A spoonful of hot (90°C) water is placed in a room whose temperature is 10°C. In one minute its temperature drops to 50°C. How many more minutes will it take for its temperature to reach 15°C?
Hint: The formula $T = T_\infty + (T_0 - T_\infty)e^{kt}$ applies.

(84) What is the area between the graph of $y = 4x^3 + 3x^2 + 2$ and the x-axis from $x = 0$ to $x = 1$?