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Derivatives-3

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$y - 14 = 17(x - 2)$

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Remember: $\dfrac{\frac{a}{b}}{\frac{c}{d}} = \left(\frac{a}{b}\right)\left(\frac{d}{c}\right)$

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whenever the limit exists.

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In other words, if the derivative exists, we have

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