

Tangent Lines

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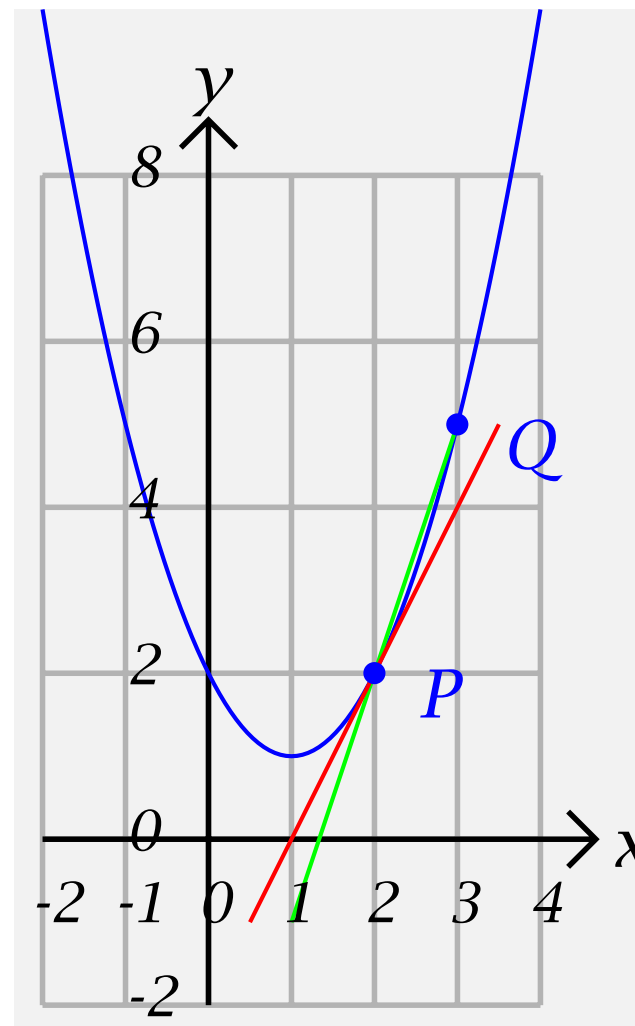
In geometry, the tangent line to a circle with centre **O** at a point **A** on the circle is defined to be the perpendicular line at **A** to the line **OA**.

The tangent lines have the special property that they intersect the circle in exactly one point. Any curve that can be obtained from a circle by **projection** (casting shadows) has tangent lines that are the projections (shadows) of the tangent lines to the circle. These curves are ellipses, parabolas, and hyperbolas, and their tangents have the same intersection property.

If we wish to define tangent lines to other curves, we cannot expect the single intersection property to still hold true. If we are given the graph of a function with equation $y = f(x)$, and wish to define the tangent to the graph at a point $\mathbf{P} = (a, f(a))$ on the graph, then all we need is to define its slope m , and we can use the point-slope form $y - f(a) = m(x - a)$

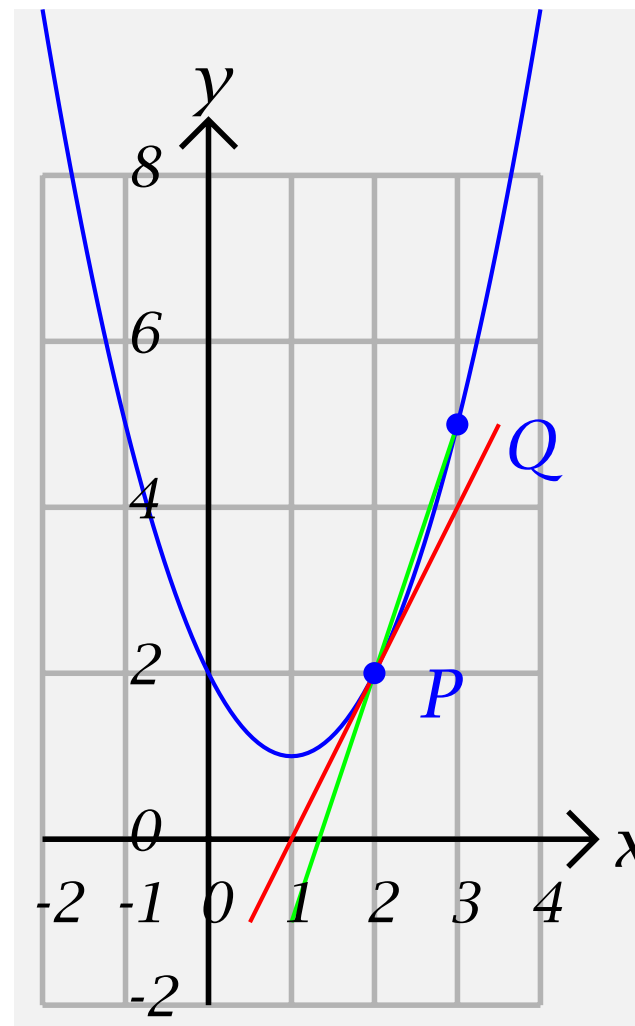
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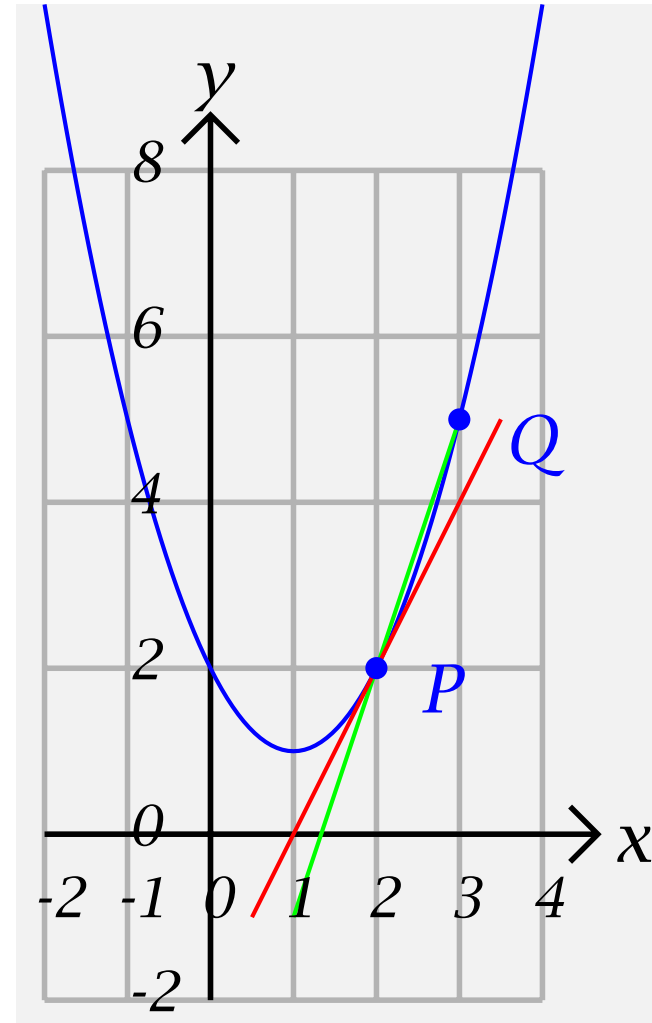
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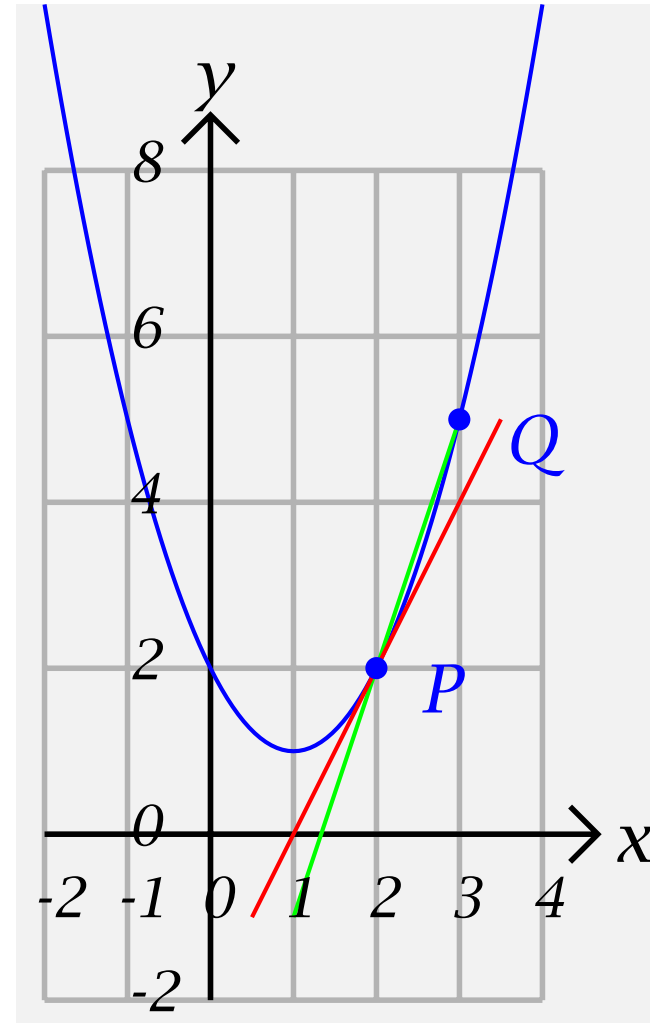
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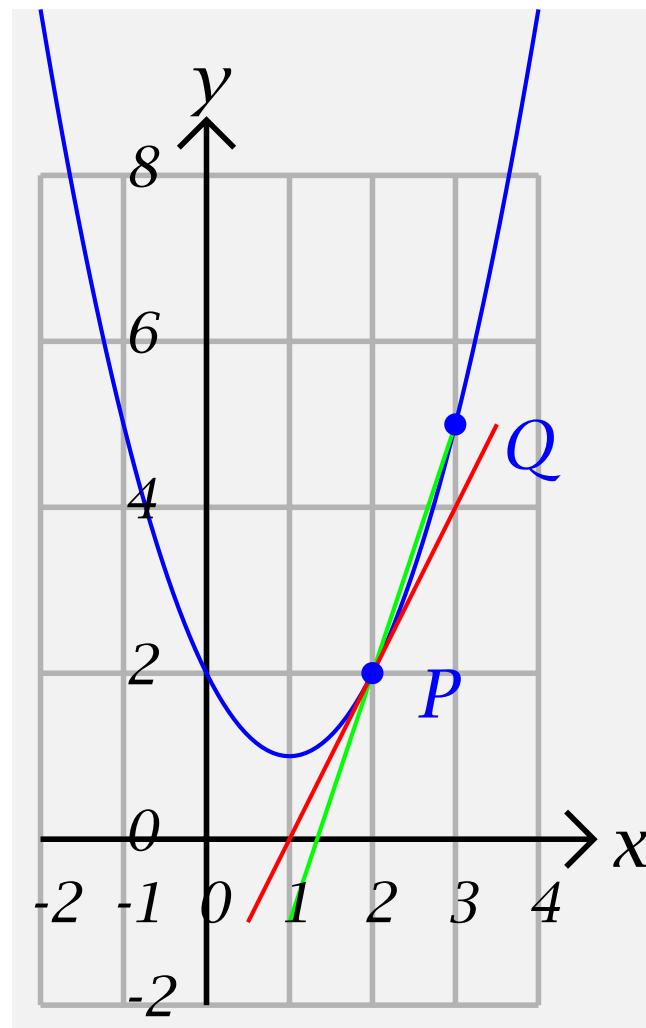
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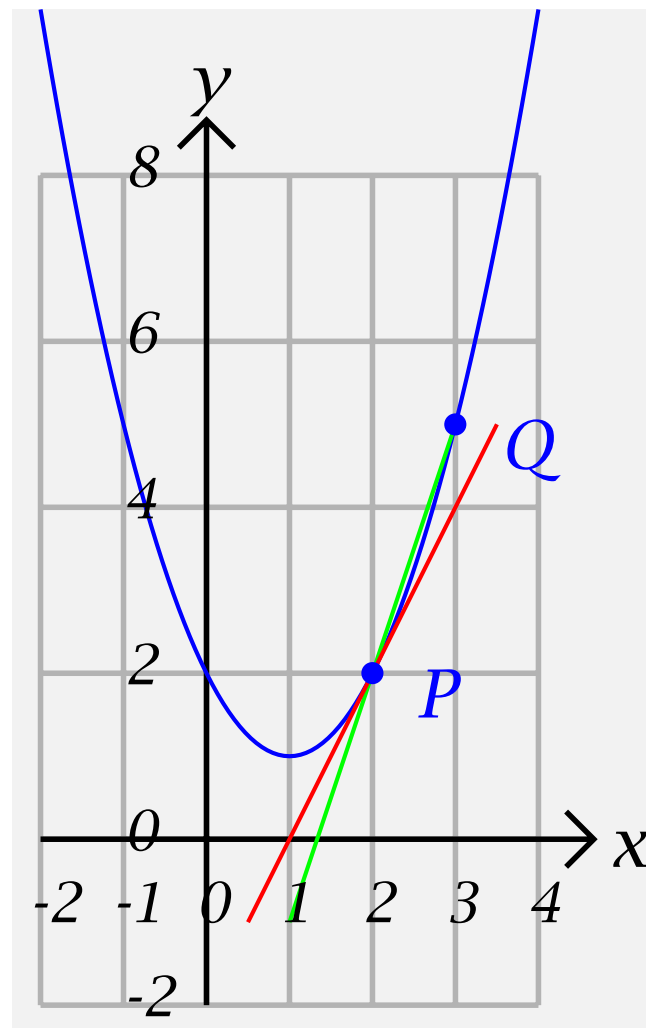
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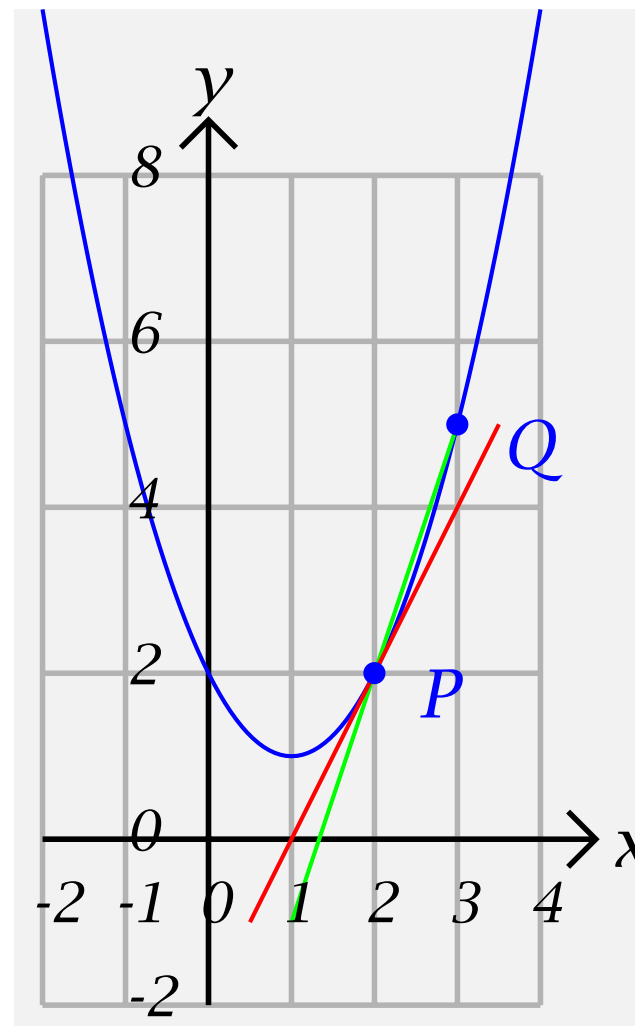
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we define the **tangent line** to $y = f(x)$ at P to be the line through P with slope m .



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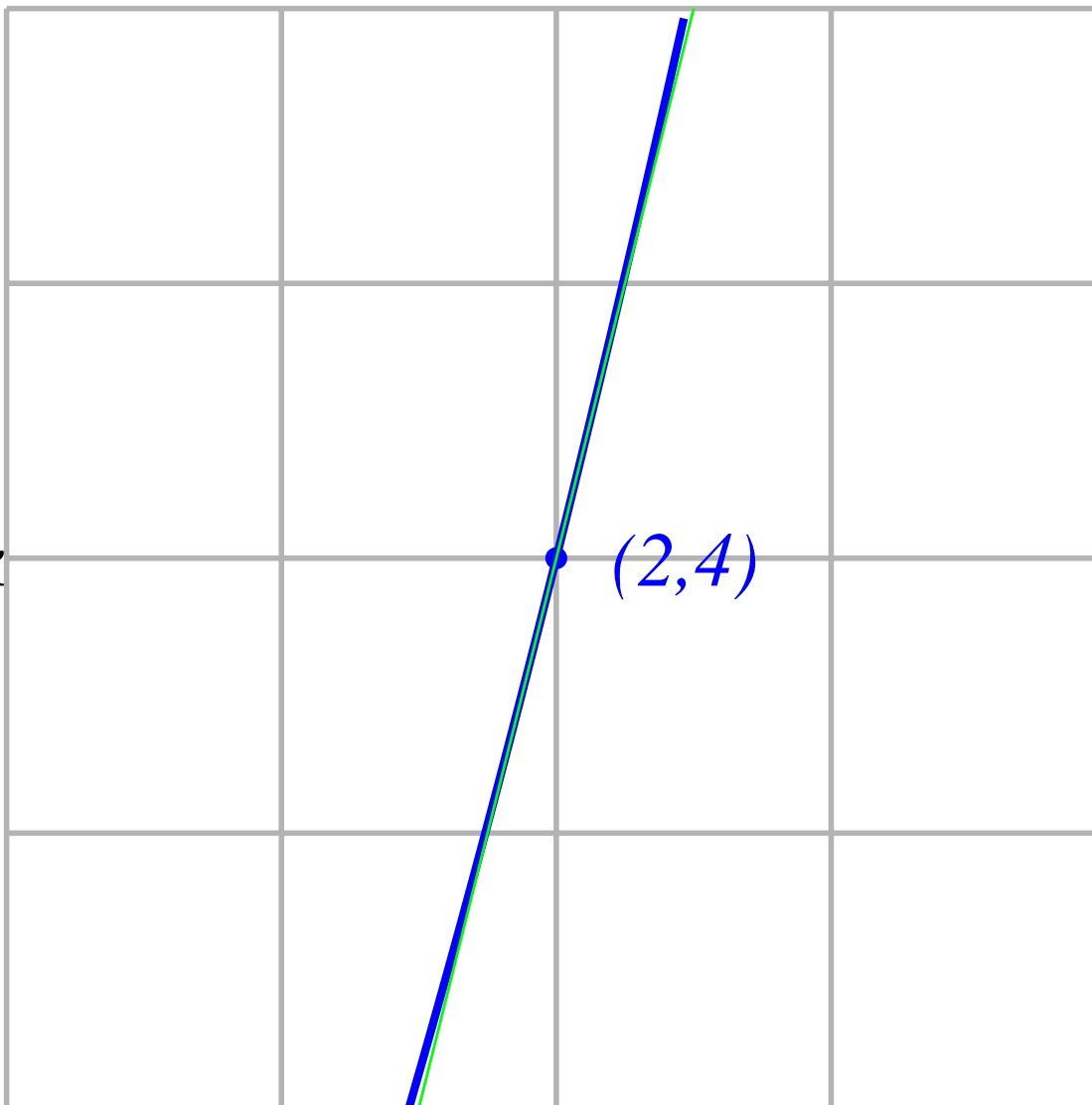
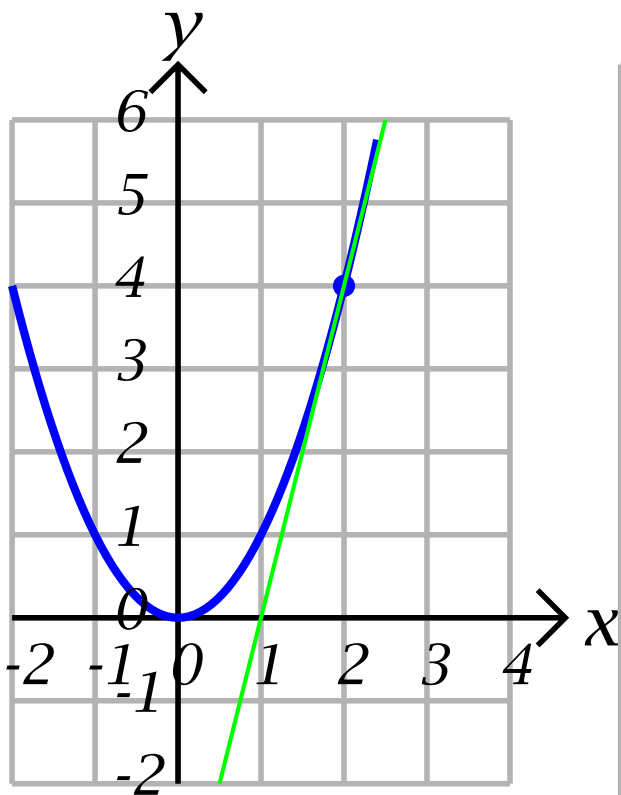
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Tangent Lines-12

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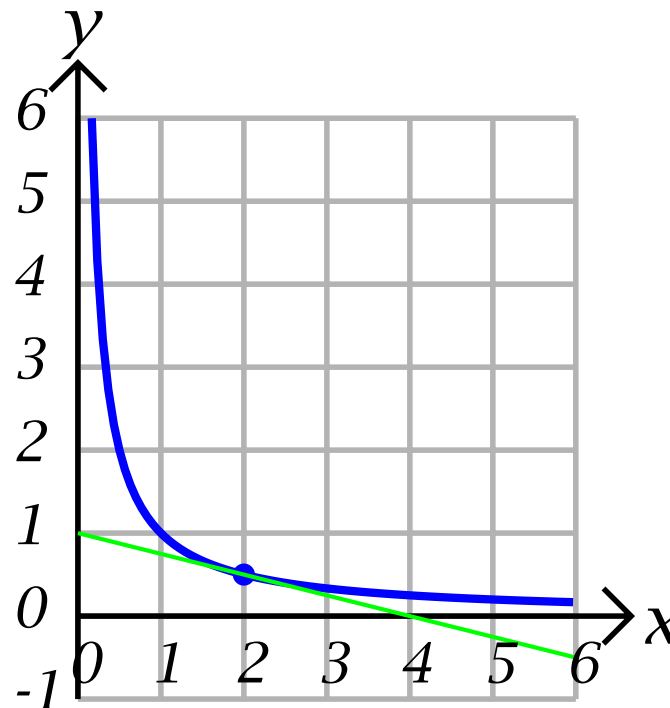
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Tangent Lines-15

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Tangent Lines-15

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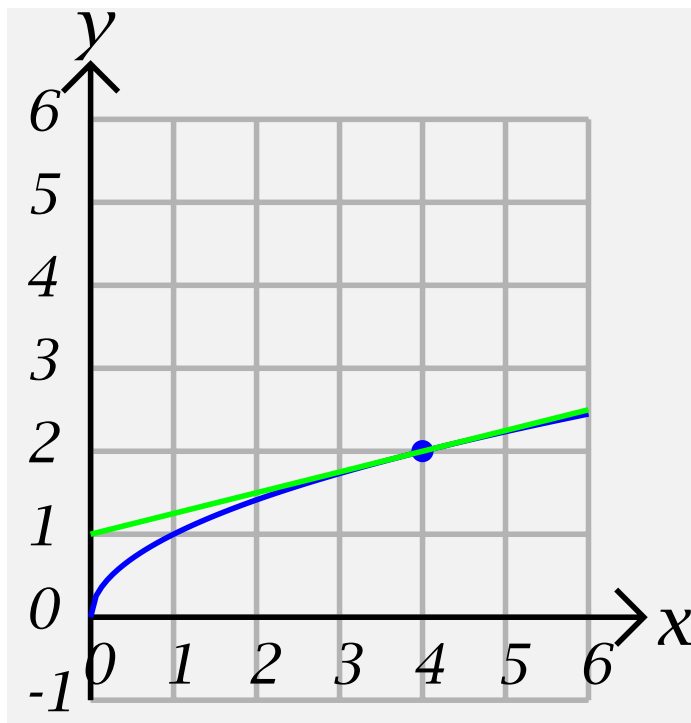
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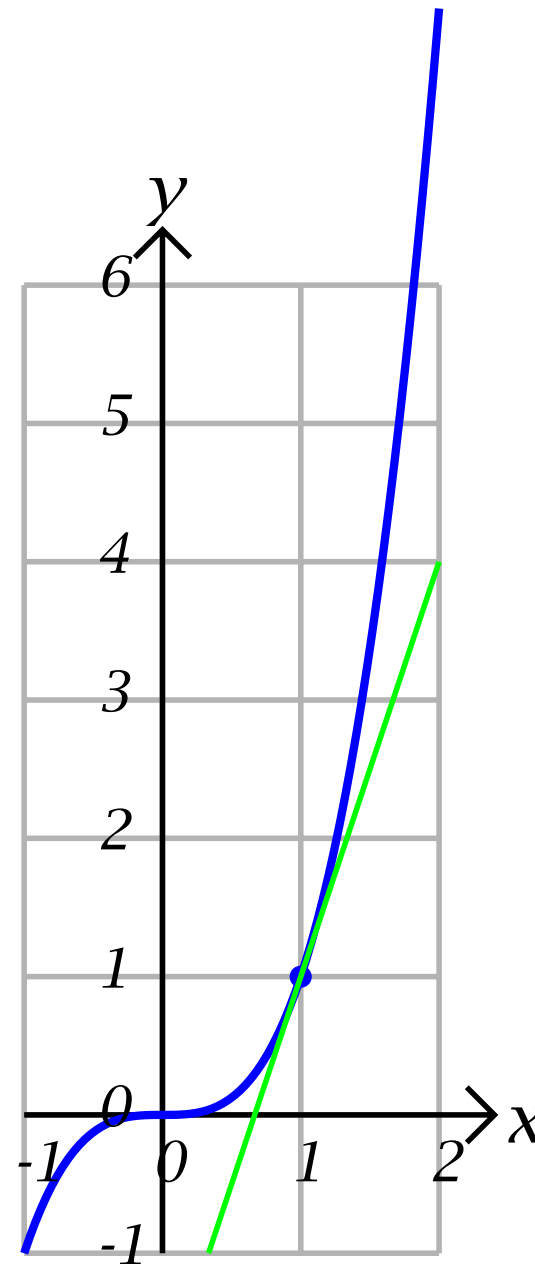
$$\frac{1}{(2+2)} =$$

$$\frac{1}{4}$$



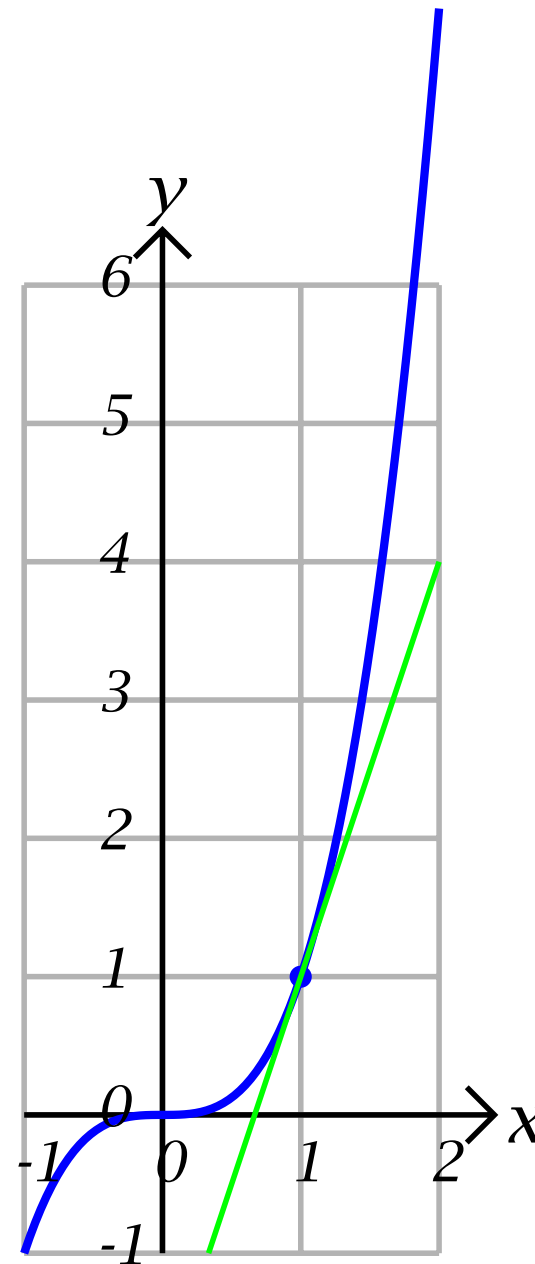
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$$m_{PQ} = \frac{f(b) - f(a)}{b - a} =$$



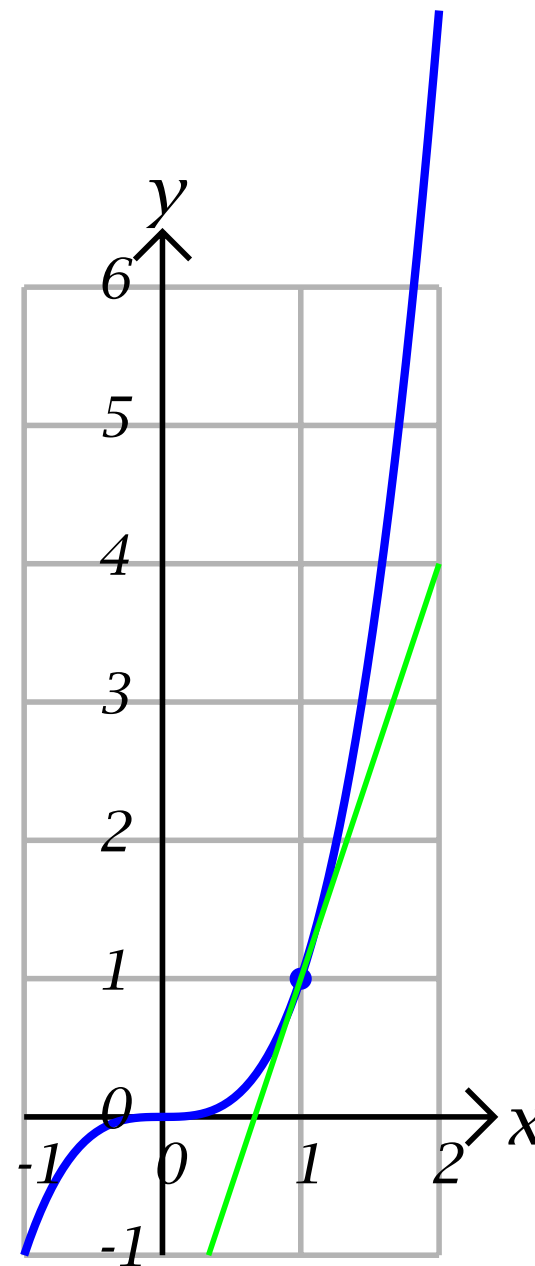
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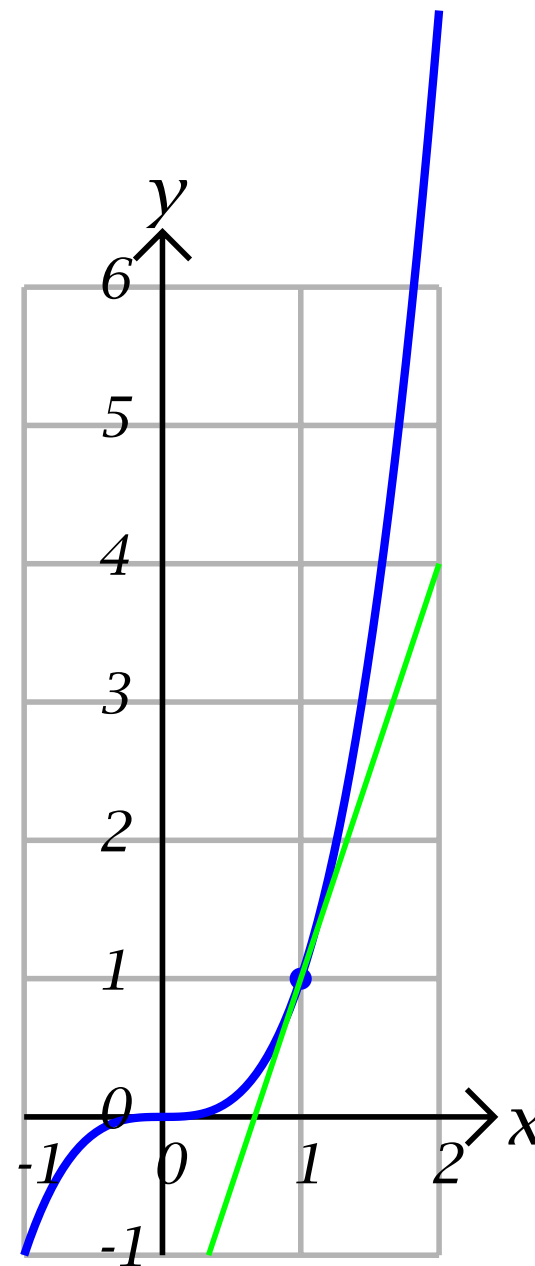
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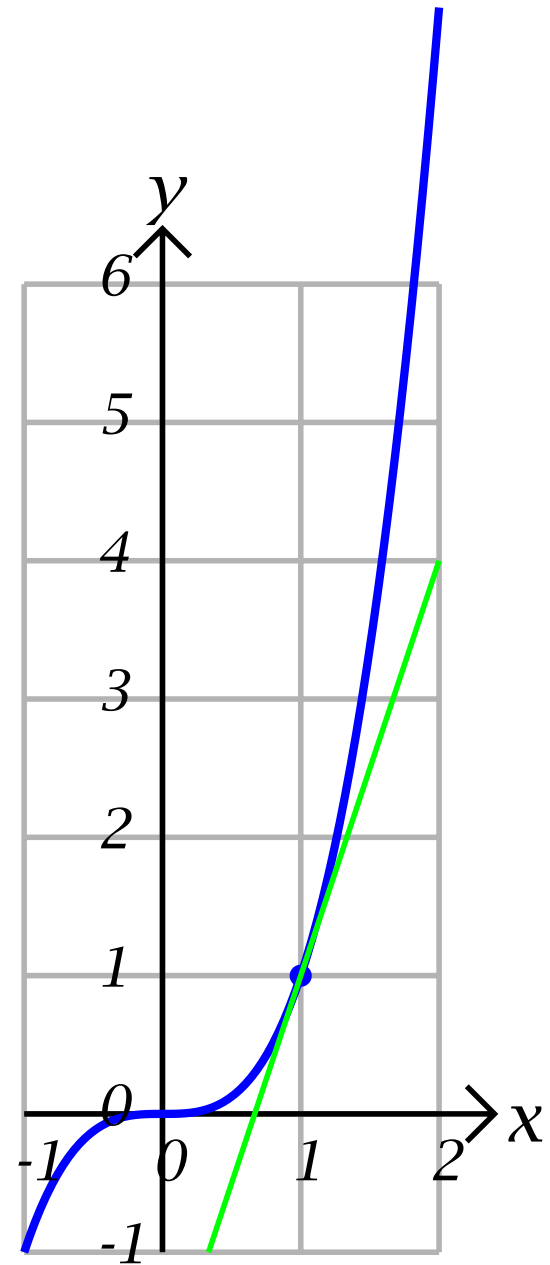
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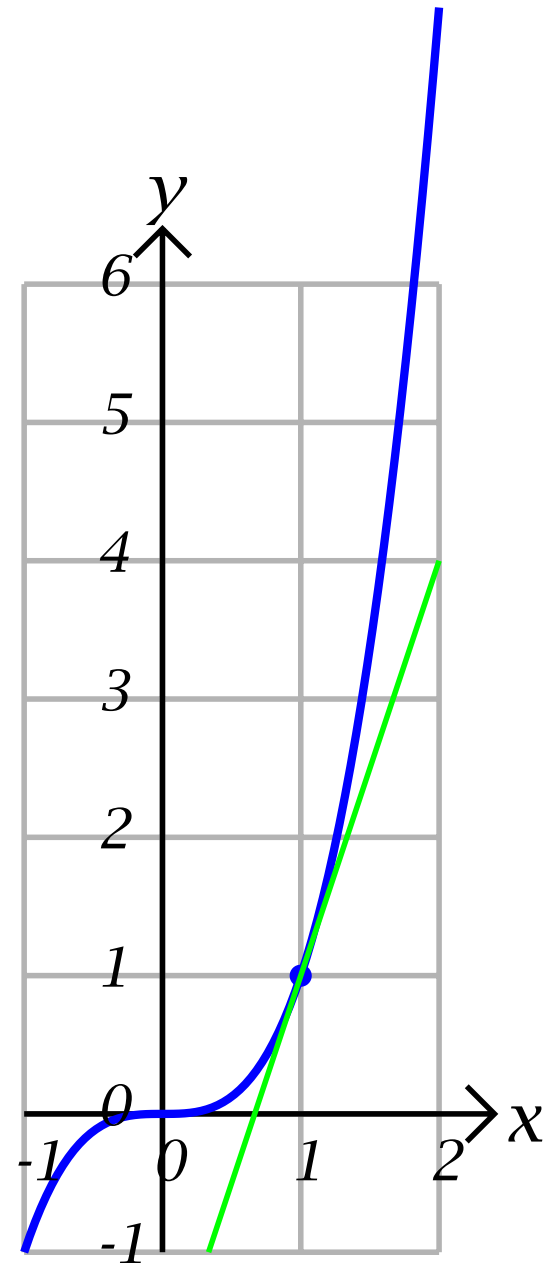
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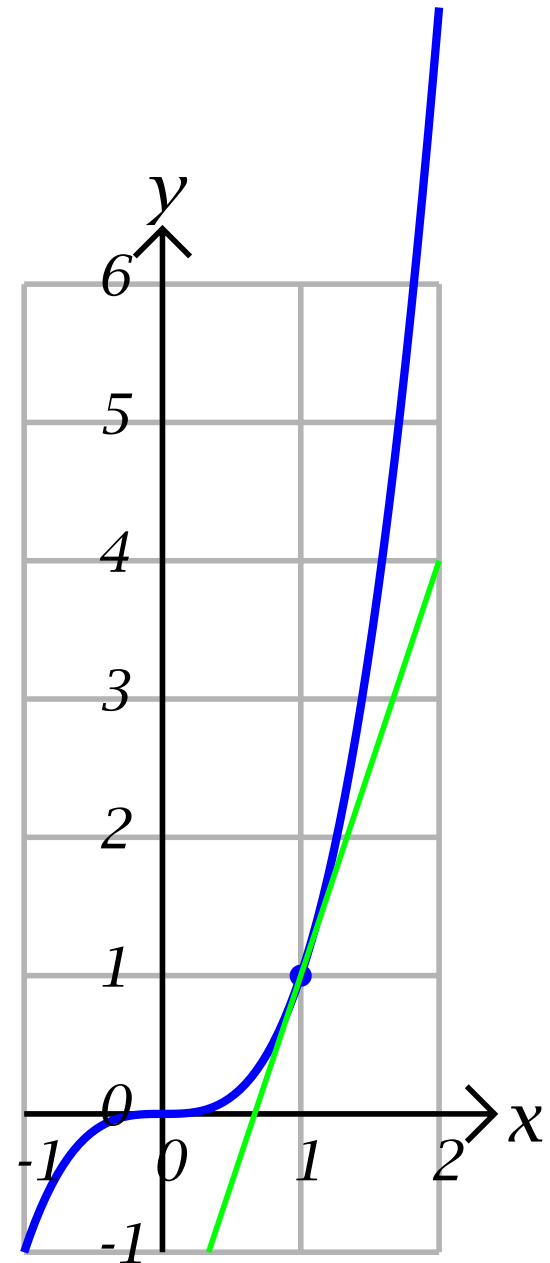
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$$y - 1 = 3(x - 1)$$



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$$\frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

Example: $s(t) = 2t^2 - 6t + 3$

Time Interval = $[t_1, t_2]$	Average Velocity = $\frac{s(t_2) - s(t_1)}{t_2 - t_1}$
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$[3, 4]$	$\frac{s(4) - s(3)}{4 - 3} = \frac{11 - 3}{1} = 8$

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$$\begin{aligned} \frac{s(b) - s(a)}{b - a} &= \frac{2b^2 - 6b + 3 - (2a^2 - 6a + 3)}{b - a} = \\ &= \frac{2(b^2 - a^2) - 6(b - a)}{b - a} = \end{aligned}$$

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$$\frac{2(b - a)(b + a) - 6(b - a)}{b - a} = 2(b + a) - 6$$

Let us look at the Average Velocity near $t_1 = 3$:

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$[3, 3.0001]$	$2(3 + 3.0001) - 6 = 6.0002$
$[3, 3.00001]$	$2(3 + 3.00001) - 6 = 6.00002$

and

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Time Interval = $[t_1, 3]$	Average Velocity = $2(t_1 + 3) - 6$
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and

Time Interval = $[t_1, 3]$	Average Velocity = $2(t_1 + 3) - 6$
$[2, 3]$	$2(2 + 3) - 6 = 4$
$[2.9, 3]$	$2(2.9 + 3) - 6 = 5.8$

and

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$[2, 3]$	$2(2 + 3) - 6 = 4$
$[2.9, 3]$	$2(2.9 + 3) - 6 = 5.8$
$[2.99, 3]$	$2(2.99 + 3) - 6 = 5.98$

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$[2.999, 3]$	$2(2.999 + 3) - 6 = 5.998$

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$[2.9999, 3]$	$2(2.9999 + 3) - 6 = 5.9998$
$[2.99999, 3]$	$2(2.99999 + 3) - 6 = 5.99998$

If we take the limit as t approaches 3, we get:

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This limit is called the **Instantaneous Velocity** at time $t = 3$.

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$$\lim_{h \rightarrow 0} \frac{2(t^2 + 2th + h^2) - 6t - 6h + 3 - 2t^2 + 6t - 3}{h} =$$

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