The Chain Rule

Recall that the

The Chain Rule

Recall that the composite function or

The Chain Rule

Recall that the composite function or composition of two functions is the function obtained by applying them one after the other.

The Chain Rule

Recall that the composite function or composition of two functions is the function obtained by applying them one after the other.

For example, if $f(x) = \dfrac{1}{x}$ and $g(x) = x^3 + 2$, then

The Chain Rule

Recall that the composite function or composition of two functions is the function obtained by applying them one after the other.

For example, if $f(x) = \dfrac{1}{x}$ and $g(x) = x^3 + 2$, then

$f(g(x)) =$

The Chain Rule

Recall that the composite function or composition of two functions is the function obtained by applying them one after the other.

For example, if $f(x) = \dfrac{1}{x}$ and $g(x) = x^3 + 2$, then

$$f(g(x)) = \frac{1}{g(x)} =$$

The Chain Rule

Recall that the composite function or composition of two functions is the function obtained by applying them one after the other.

For example, if $f(x) = \frac{1}{x}$ and $g(x) = x^3 + 2$, then

$$f(g(x)) = \frac{1}{g(x)} = \frac{1}{x^3 + 2}$$

The Chain Rule

Recall that the composite function or composition of two functions is the function obtained by applying them one after the other.

For example, if $f(x) = \dfrac{1}{x}$ and $g(x) = x^3 + 2$, then

$$f(g(x)) = \frac{1}{g(x)} = \frac{1}{x^3 + 2}$$

and $g(f(x)) =$

The Chain Rule

Recall that the composite function or composition of two functions is the function obtained by applying them one after the other.

For example, if $f(x) = \frac{1}{x}$ and $g(x) = x^3 + 2$, then

$$f(g(x)) = \frac{1}{g(x)} = \frac{1}{x^3 + 2}$$

and $g(f(x)) = (f(x))^3 + 2 =$

The Chain Rule

Recall that the composite function or composition of two functions is the function obtained by applying them one after the other.

For example, if $f(x) = \dfrac{1}{x}$ and $g(x) = x^3 + 2$, then

$$f(g(x)) = \frac{1}{g(x)} = \frac{1}{x^3 + 2}$$

and $g(f(x)) = (f(x))^3 + 2 = \left(\dfrac{1}{x}\right)^3 + 2 =$

The Chain Rule

Recall that the composite function or composition of two functions is the function obtained by applying them one after the other.

For example, if $f(x) = \dfrac{1}{x}$ and $g(x) = x^3 + 2$, then

$$f(g(x)) = \frac{1}{g(x)} = \frac{1}{x^3 + 2}$$

and $g(f(x)) = (f(x))^3 + 2 = \left(\dfrac{1}{x}\right)^3 + 2 = \dfrac{1}{x^3} + 2$

It is important for the student to be able to recognize that a function is the composition of two or more functions, so that the upcoming “Chain Rule” for differentiating composites may be used.

It is important for the student to be able to recognize that a function is the composition of two or more functions, so that the upcoming “Chain Rule” for differentiating composites may be used.

For example, if $h(x) = \dfrac{1}{x^3 + 2}$, we see that we can let

$u = g(x) =$

It is important for the student to be able to recognize that a function is the composition of two or more functions, so that the upcoming “Chain Rule” for differentiating composites may be used.

For example, if $h(x) = \dfrac{1}{x^3 + 2}$, we see that we can let $u = g(x) = x^3 + 2$ to get

It is important for the student to be able to recognize that a function is the composition of two or more functions, so that the upcoming "Chain Rule" for differentiating composites may be used.

For example, if $h(x) = \dfrac{1}{x^3 + 2}$, we see that we can let $u = g(x) = x^3 + 2$ to get $h(x) = \dfrac{1}{u} = \dfrac{1}{g(x)}$.

It is important for the student to be able to recognize that a function is the composition of two or more functions, so that the upcoming "Chain Rule" for differentiating composites may be used.

For example, if $h(x) = \dfrac{1}{x^3 + 2}$, we see that we can let $u = g(x) = x^3 + 2$ to get $h(x) = \dfrac{1}{u} = \dfrac{1}{g(x)}$.

Then we let $f(x) = \dfrac{1}{x}$,

It is important for the student to be able to recognize that a function is the composition of two or more functions, so that the upcoming “Chain Rule” for differentiating composites may be used.

For example, if $h(x) = \frac{1}{x^3 + 2}$, we see that we can let $u = g(x) = x^3 + 2$ to get $h(x) = \frac{1}{u} = \frac{1}{g(x)}$.

Then we let $f(x) = \frac{1}{x}$, so that $h(x) = f(u) = f(g(x))$

The derivative of the composition of two non-constant functions is equal to the product of their derivatives, evaluated appropriately.

The derivative of the composition of two non-constant functions is equal to the product of their derivatives, evaluated appropriately.

We have, in function notation,

The derivative of the composition of two non-constant functions is equal to the product of their derivatives, evaluated appropriately.

We have, in function notation,

The derivative of the composition of two non-constant functions is equal to the product of their derivatives, evaluated appropriately.

We have, in function notation,

$$\left(g(h(x))\right)' = g'(h(x))h'(x)$$

The derivative of the composition of two non-constant functions is equal to the product of their derivatives, evaluated appropriately.

We have, in function notation,

$$\left(g(h(x))\right)' = g'(h(x))h'(x)$$

or, in Leibnitz notation, if $y = f(g(x))$ and $u = g(x)$,

The derivative of the composition of two non-constant functions is equal to the product of their derivatives, evaluated appropriately.

We have, in function notation,

$$\left(g(h(x))\right)' = g'(h(x))h'(x)$$

or, in Leibnitz notation, if $y = f(g(x))$ and $u = g(x)$,

The derivative of the composition of two non-constant functions is equal to the product of their derivatives, evaluated appropriately.

We have, in function notation,

$$\big(g(h(x))\big)' = g'(h(x))h'(x)$$

or, in Leibniz notation, if $y = f(g(x))$ and $u = g(x)$,

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

The derivative of the composition of two non-constant functions is equal to the product of their derivatives, evaluated appropriately.

We have, in function notation,

$$\left(g(h(x))\right)' = g'(h(x))h'(x)$$

or, in Leibniz notation, if $y = f(g(x))$ and $u = g(x)$,

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

We may also write

The derivative of the composition of two non-constant functions is equal to the product of their derivatives, evaluated appropriately.

We have, in function notation,

$$(g(h(x)))' = g'(h(x))h'(x)$$

or, in Leibniz notation, if $y = f(g(x))$ and $u = g(x)$,

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

We may also write

$$\frac{d}{dx}[f(u)] = f'(u)\frac{du}{dx}$$

Example 1: Using $g(x) = \dfrac{1}{x} = x^{-1}$ and $h(x) = x^3 + 2$,

we have

Example 1: Using $g(x) = \frac{1}{x} = x^{-1}$ and $h(x) = x^3 + 2$,

we have $g'(x) = (-1)x^{-2}$ and

Example 1: Using $g(x) = \frac{1}{x} = x^{-1}$ and $h(x) = x^3 + 2$,

we have $g'(x) = (-1)x^{-2}$ and $h'(x) = 3x^2$,

Example 1: Using $g(x) = \frac{1}{x} = x^{-1}$ and $h(x) = x^3 + 2$,

we have $g'(x) = (-1)x^{-2}$ and $h'(x) = 3x^2$,

$g'(h(x)) =$

Example 1: Using $g(x) = \frac{1}{x} = x^{-1}$ and $h(x) = x^3 + 2$,

we have $g'(x) = (-1)x^{-2}$ and $h'(x) = 3x^2$,

$g'(h(x)) = (-1)(h(x))^{-2}$,

Example 1: Using $g(x) = \frac{1}{x} = x^{-1}$ and $h(x) = x^3 + 2$,

we have $g'(x) = (-1)x^{-2}$ and $h'(x) = 3x^2$,

$g'(h(x)) = (-1)(h(x))^{-2}$, so we get

$$\left(\frac{1}{x^3 + 2}\right)' =$$

Example 1: Using $g(x) = \frac{1}{x} = x^{-1}$ and $h(x) = x^3 + 2$,

we have $g'(x) = (-1)x^{-2}$ and $h'(x) = 3x^2$,

$g'(h(x)) = (-1)(h(x))^{-2}$, so we get

$$\left(\frac{1}{x^3 + 2}\right)' = g'(h(x))h'(x) =$$

Example 1: Using $g(x) = \frac{1}{x} = x^{-1}$ and $h(x) = x^3 + 2$,

we have $g'(x) = (-1)x^{-2}$ and $h'(x) = 3x^2$,

$g'(h(x)) = (-1)(h(x))^{-2}$, so we get

$$\left(\frac{1}{x^3+2}\right)' = g'(h(x))h'(x) = (-1)(h(x))^{-2}(3x^2) =$$

Example 1: Using $g(x) = \frac{1}{x} = x^{-1}$ and $h(x) = x^3 + 2$,

we have $g'(x) = (-1)x^{-2}$ and $h'(x) = 3x^2$,

$g'(h(x)) = (-1)(h(x))^{-2}$, so we get

$$\left(\frac{1}{x^3 + 2}\right)' = g'(h(x))h'(x) = (-1)(h(x))^{-2}(3x^2) =$$

$$(-1)(x^3 + 2)^{-2}(3x^2) =$$

Example 1: Using $g(x) = \frac{1}{x} = x^{-1}$ and $h(x) = x^3 + 2$,

we have $g'(x) = (-1)x^{-2}$ and $h'(x) = 3x^2$,

$g'(h(x)) = (-1)(h(x))^{-2}$, so we get

$$\left(\frac{1}{x^3+2}\right)' = g'(h(x))h'(x) = (-1)(h(x))^{-2}(3x^2) =$$

$$(-1)(x^3+2)^{-2}(3x^2) =$$

Example 1: Using $g(x) = \frac{1}{x} = x^{-1}$ and $h(x) = x^3 + 2$,

we have $g'(x) = (-1)x^{-2}$ and $h'(x) = 3x^2$,

$g'(h(x)) = (-1)(h(x))^{-2}$, so we get

$$\left(\frac{1}{x^3+2}\right)' = g'(h(x))h'(x) = (-1)(h(x))^{-2}(3x^2) =$$

$$(-1)(x^3+2)^{-2}(3x^2) = \frac{-3x^2}{(x^3+2)^2}$$

Example 2: Let $g(x) = x^3$, and $h(x) = x^2$, so that

Example 2: Let $g(x) = x^3$, and $h(x) = x^2$, so that

$$g(h(x)) =$$

Example 2: Let $g(x) = x^3$, and $h(x) = x^2$, so that

$$g(h(x)) = (h(x))^3 =$$

Example 2: Let $g(x) = x^3$, and $h(x) = x^2$, so that

$$g(h(x)) = (h(x))^3 = (x^2)^3 =$$

Example 2: Let $g(x) = x^3$, and $h(x) = x^2$, so that

$$g(h(x)) = (h(x))^3 = (x^2)^3 = x^6.$$

Then $g'(x) = 3x^2$, so

Example 2: Let $g(x) = x^3$, and $h(x) = x^2$, so that

$g(h(x)) = (h(x))^3 = (x^2)^3 = x^6$.

Then $g'(x) = 3x^2$, so $g'(h(x)) =$

Example 2: Let $g(x) = x^3$, and $h(x) = x^2$, so that

$$g(h(x)) = (h(x))^3 = (x^2)^3 = x^6.$$

Then $g'(x) = 3x^2$, so $g'(h(x)) = 3(h(x))^2$, and

Example 2: Let $g(x) = x^3$, and $h(x) = x^2$, so that

$g(h(x)) = (h(x))^3 = (x^2)^3 = x^6$.

Then $g'(x) = 3x^2$, so $g'(h(x)) = 3(h(x))^2$, and $h'(x) =$

Example 2: Let $g(x) = x^3$, and $h(x) = x^2$, so that

$g(h(x)) = (h(x))^3 = (x^2)^3 = x^6$.

Then $g'(x) = 3x^2$, so $g'(h(x)) = 3(h(x))^2$, and $h'(x) = 2x$,

Example 2: Let $g(x) = x^3$, and $h(x) = x^2$, so that

$g(h(x)) = (h(x))^3 = (x^2)^3 = x^6$.

Then $g'(x) = 3x^2$, so $g'(h(x)) = 3(h(x))^2$, and $h'(x) = 2x$,

so the Chain Rule gives us

Example 2: Let $g(x) = x^3$, and $h(x) = x^2$, so that

$g(h(x)) = (h(x))^3 = (x^2)^3 = x^6$.

Then $g'(x) = 3x^2$, so $g'(h(x)) = 3(h(x))^2$, and $h'(x) = 2x$,

so the Chain Rule gives us

$\big(g(h(x))\big)' =$

Example 2: Let $g(x) = x^3$, and $h(x) = x^2$, so that

$$g(h(x)) = (h(x))^3 = (x^2)^3 = x^6.$$

Then $g'(x) = 3x^2$, so $g'(h(x)) = 3(h(x))^2$, and $h'(x) = 2x$,

so the Chain Rule gives us

$$\big(g(h(x))\big)' = g'(h(x))h'(x) =$$

Example 2: Let $g(x) = x^3$, and $h(x) = x^2$, so that

$g(h(x)) = (h(x))^3 = (x^2)^3 = x^6$.

Then $g'(x) = 3x^2$, so $g'(h(x)) = 3(h(x))^2$, and $h'(x) = 2x$,

so the Chain Rule gives us

$$\left(g(h(x))\right)' = g'(h(x))h'(x) = \left(3(h(x))^2\right)(2x) =$$

Example 2: Let $g(x) = x^3$, and $h(x) = x^2$, so that

$g(h(x)) = (h(x))^3 = (x^2)^3 = x^6$.

Then $g'(x) = 3x^2$, so $g'(h(x)) = 3(h(x))^2$, and $h'(x) = 2x$,

so the Chain Rule gives us

$(g(h(x)))' = g'(h(x))h'(x) = \left(3(h(x))^2\right)(2x) =$

$\left(3(x^2)^2\right)(2x) =$

Example 2: Let $g(x) = x^3$, and $h(x) = x^2$, so that

$g(h(x)) = (h(x))^3 = (x^2)^3 = x^6$.

Then $g'(x) = 3x^2$, so $g'(h(x)) = 3(h(x))^2$, and $h'(x) = 2x$,

so the Chain Rule gives us

$$(g(h(x)))' = g'(h(x))h'(x) = \left(3(h(x))^2\right)(2x) =$$

$$\left(3(x^2)^2\right)(2x) = \left(3x^4\right)(2x) =$$

Example 2: Let $g(x) = x^3$, and $h(x) = x^2$, so that

$$g(h(x)) = (h(x))^3 = (x^2)^3 = x^6.$$

Then $g'(x) = 3x^2$, so $g'(h(x)) = 3(h(x))^2$, and $h'(x) = 2x$,

so the Chain Rule gives us

$$(g(h(x)))' = g'(h(x))h'(x) = \left(3(h(x))^2\right)(2x) =$$

$$\left(3(x^2)^2\right)(2x) = \left(3x^4\right)(2x) =$$

Example 2: Let $g(x) = x^3$, and $h(x) = x^2$, so that

$g(h(x)) = (h(x))^3 = (x^2)^3 = x^6$.

Then $g'(x) = 3x^2$, so $g'(h(x)) = 3(h(x))^2$, and $h'(x) = 2x$,

so the Chain Rule gives us

$$(g(h(x)))' = g'(h(x))h'(x) = \left(3(h(x))^2\right)(2x) =$$

$$\left(3(x^2)^2\right)(2x) = \left(3x^4\right)(2x) = \boxed{6x^5}$$, as expected.

Example 3: Let $g(x) = x^3 + 3$, and $h(x) = x^2 + 2$, so that

Example 3: Let $g(x) = x^3 + 3$, and $h(x) = x^2 + 2$, so that

$g(h(x)) =$

Example 3: Let $g(x) = x^3 + 3$, and $h(x) = x^2 + 2$, so that

$$g(h(x)) = (h(x))^3 + 3 =$$

Example 3: Let $g(x) = x^3 + 3$, and $h(x) = x^2 + 2$, so that

$$g(h(x)) = (h(x))^3 + 3 = (x^2 + 2)^3 + 3.$$

Example 3: Let $g(x) = x^3 + 3$, and $h(x) = x^2 + 2$, so that

$$g(h(x)) = (h(x))^3 + 3 = (x^2 + 2)^3 + 3.$$

Then $g'(x) = 3x^2$,

Example 3: Let $g(x) = x^3 + 3$, and $h(x) = x^2 + 2$, so that

$g(h(x)) = (h(x))^3 + 3 = (x^2 + 2)^3 + 3.$

Then $g'(x) = 3x^2$, so $g'(h(x)) =$

Example 3: Let $g(x) = x^3 + 3$, and $h(x) = x^2 + 2$, so that

$g(h(x)) = (h(x))^3 + 3 = (x^2 + 2)^3 + 3.$

Then $g'(x) = 3x^2$, so $g'(h(x)) = 3(h(x))^2$, and

Example 3: Let $g(x) = x^3 + 3$, and $h(x) = x^2 + 2$, so that

$$g(h(x)) = (h(x))^3 + 3 = (x^2 + 2)^3 + 3.$$

Then $g'(x) = 3x^2$, so $g'(h(x)) = 3(h(x))^2$, and $h'(x) = 2x$,

Example 3: Let $g(x) = x^3 + 3$, and $h(x) = x^2 + 2$, so that

$g(h(x)) = (h(x))^3 + 3 = (x^2 + 2)^3 + 3.$

Then $g'(x) = 3x^2$, so $g'(h(x)) = 3(h(x))^2$, and $h'(x) = 2x$,

so the Chain Rule gives us

Example 3: Let $g(x) = x^3 + 3$, and $h(x) = x^2 + 2$, so that

$g(h(x)) = (h(x))^3 + 3 = (x^2 + 2)^3 + 3.$

Then $g'(x) = 3x^2$, so $g'(h(x)) = 3(h(x))^2$, and $h'(x) = 2x$,

so the Chain Rule gives us

$\big(g(h(x))\big)' =$

Example 3: Let $g(x) = x^3 + 3$, and $h(x) = x^2 + 2$, so that

$g(h(x)) = (h(x))^3 + 3 = (x^2 + 2)^3 + 3$.

Then $g'(x) = 3x^2$, so $g'(h(x)) = 3(h(x))^2$, and $h'(x) = 2x$,

so the Chain Rule gives us

$$(g(h(x)))' = g'(h(x))h'(x) = \left(3(h(x))^2\right)(2x) =$$

Example 3: Let $g(x) = x^3 + 3$, and $h(x) = x^2 + 2$, so that

$g(h(x)) = (h(x))^3 + 3 = (x^2 + 2)^3 + 3.$

Then $g'(x) = 3x^2$, so $g'(h(x)) = 3(h(x))^2$, and $h'(x) = 2x$,

so the Chain Rule gives us

$$(g(h(x)))' = g'(h(x))h'(x) = \left(3(h(x))^2\right)(2x) =$$
$$\left(3(x^2 + 2)^2\right)(2x) =$$

Example 3: Let $g(x) = x^3 + 3$, and $h(x) = x^2 + 2$, so that

$$g(h(x)) = (h(x))^3 + 3 = (x^2 + 2)^3 + 3.$$

Then $g'(x) = 3x^2$, so $g'(h(x)) = 3(h(x))^2$, and $h'(x) = 2x$,

so the Chain Rule gives us

$$(g(h(x)))' = g'(h(x))h'(x) = \left(3(h(x))^2\right)(2x) =$$
$$\left(3(x^2 + 2)^2\right)(2x) =$$

Example 3: Let $g(x) = x^3 + 3$, and $h(x) = x^2 + 2$, so that

$g(h(x)) = (h(x))^3 + 3 = (x^2 + 2)^3 + 3.$

Then $g'(x) = 3x^2$, so $g'(h(x)) = 3(h(x))^2$, and $h'(x) = 2x$,

so the Chain Rule gives us

$$(g(h(x)))' = g'(h(x))h'(x) = \left(3(h(x))^2\right)(2x) =$$
$$\left(3(x^2 + 2)^2\right)(2x) =$$

$6x(x^2 + 2)^2$

Example 4: Find $f'(x)$ if $f(x) = \sqrt[3]{x^4 + x^2 + 1}$.

Example 4: Find $f'(x)$ if $f(x) = \sqrt[3]{x^4 + x^2 + 1}$.

We let $g(x) = x^{\frac{1}{3}}$ and

Example 4: Find $f'(x)$ if $f(x) = \sqrt[3]{x^4 + x^2 + 1}$.

We let $g(x) = x^{\frac{1}{3}}$ and $u = x^4 + x^2 + 1$ so that

Example 4: Find $f'(x)$ if $f(x) = \sqrt[3]{x^4 + x^2 + 1}$.

We let $g(x) = x^{\frac{1}{3}}$ and $u = x^4 + x^2 + 1$ so that $f(x) = g(u)$.

Example 4: Find $f'(x)$ if $f(x) = \sqrt[3]{x^4 + x^2 + 1}$.

We let $g(x) = x^{\frac{1}{3}}$ and $u = x^4 + x^2 + 1$ so that $f(x) = g(u)$.

Then $g'(x) =$

Example 4: Find $f'(x)$ if $f(x) = \sqrt[3]{x^4 + x^2 + 1}$.

We let $g(x) = x^{\frac{1}{3}}$ and $u = x^4 + x^2 + 1$ so that $f(x) = g(u)$.

Then $g'(x) = \frac{1}{3}x^{-\frac{2}{3}}$,

Example 4: Find $f'(x)$ if $f(x) = \sqrt[3]{x^4 + x^2 + 1}$.

We let $g(x) = x^{\frac{1}{3}}$ and $u = x^4 + x^2 + 1$ so that $f(x) = g(u)$.

Then $g'(x) = \frac{1}{3}x^{-\frac{2}{3}}$, $g'(u) =$

Example 4: Find $f'(x)$ if $f(x) = \sqrt[3]{x^4 + x^2 + 1}$.

We let $g(x) = x^{\frac{1}{3}}$ and $u = x^4 + x^2 + 1$ so that $f(x) = g(u)$.

Then $g'(x) = \frac{1}{3}x^{-\frac{2}{3}}$, $g'(u) = \frac{1}{3}u^{-\frac{2}{3}}$,

Example 4: Find $f'(x)$ if $f(x) = \sqrt[3]{x^4 + x^2 + 1}$.

We let $g(x) = x^{\frac{1}{3}}$ and $u = x^4 + x^2 + 1$ so that $f(x) = g(u)$.

Then $g'(x) = \frac{1}{3}x^{-\frac{2}{3}}$, $g'(u) = \frac{1}{3}u^{-\frac{2}{3}}$, and $\dfrac{du}{dx} =$

Example 4: Find $f'(x)$ if $f(x) = \sqrt[3]{x^4 + x^2 + 1}$.

We let $g(x) = x^{\frac{1}{3}}$ and $u = x^4 + x^2 + 1$ so that $f(x) = g(u)$.

Then $g'(x) = \frac{1}{3}x^{-\frac{2}{3}}$, $g'(u) = \frac{1}{3}u^{-\frac{2}{3}}$, and $\dfrac{du}{dx} = 4x^3 + 2x$,

Example 4: Find $f'(x)$ if $f(x) = \sqrt[3]{x^4 + x^2 + 1}$.

We let $g(x) = x^{\frac{1}{3}}$ and $u = x^4 + x^2 + 1$ so that $f(x) = g(u)$.

Then $g'(x) = \frac{1}{3}x^{-\frac{2}{3}}$, $g'(u) = \frac{1}{3}u^{-\frac{2}{3}}$, and $\dfrac{du}{dx} = 4x^3 + 2x$,

so we have

Example 4: Find $f'(x)$ if $f(x) = \sqrt[3]{x^4 + x^2 + 1}$.

We let $g(x) = x^{\frac{1}{3}}$ and $u = x^4 + x^2 + 1$ so that $f(x) = g(u)$.

Then $g'(x) = \frac{1}{3}x^{-\frac{2}{3}}$, $g'(u) = \frac{1}{3}u^{-\frac{2}{3}}$, and $\dfrac{du}{dx} = 4x^3 + 2x$,

so we have $f'(x) =$

Example 4: Find $f'(x)$ if $f(x) = \sqrt[3]{x^4 + x^2 + 1}$.

We let $g(x) = x^{\frac{1}{3}}$ and $u = x^4 + x^2 + 1$ so that $f(x) = g(u)$.

Then $g'(x) = \frac{1}{3}x^{-\frac{2}{3}}$, $g'(u) = \frac{1}{3}u^{-\frac{2}{3}}$, and $\dfrac{du}{dx} = 4x^3 + 2x$,

so we have $f'(x) = g'(u)\dfrac{du}{dx} =$

Example 4: Find $f'(x)$ if $f(x) = \sqrt[3]{x^4 + x^2 + 1}$.

We let $g(x) = x^{\frac{1}{3}}$ and $u = x^4 + x^2 + 1$ so that $f(x) = g(u)$.

Then $g'(x) = \frac{1}{3}x^{-\frac{2}{3}}$, $g'(u) = \frac{1}{3}u^{-\frac{2}{3}}$, and $\dfrac{du}{dx} = 4x^3 + 2x$,

so we have $f'(x) = g'(u)\dfrac{du}{dx} = \dfrac{1}{3}(u)^{-\frac{2}{3}}(4x^3 + 2x) =$

Example 4: Find $f'(x)$ if $f(x) = \sqrt[3]{x^4 + x^2 + 1}$.

We let $g(x) = x^{\frac{1}{3}}$ and $u = x^4 + x^2 + 1$ so that $f(x) = g(u)$.

Then $g'(x) = \frac{1}{3}x^{-\frac{2}{3}}$, $g'(u) = \frac{1}{3}u^{-\frac{2}{3}}$, and $\frac{du}{dx} = 4x^3 + 2x$,

so we have $f'(x) = g'(u)\frac{du}{dx} = \frac{1}{3}(u)^{-\frac{2}{3}}(4x^3 + 2x) =$

Example 4: Find $f'(x)$ if $f(x) = \sqrt[3]{x^4 + x^2 + 1}$.

We let $g(x) = x^{\frac{1}{3}}$ and $u = x^4 + x^2 + 1$ so that $f(x) = g(u)$.

Then $g'(x) = \frac{1}{3}x^{-\frac{2}{3}}$, $g'(u) = \frac{1}{3}u^{-\frac{2}{3}}$, and $\frac{du}{dx} = 4x^3 + 2x$,

so we have $f'(x) = g'(u)\frac{du}{dx} = \frac{1}{3}(u)^{-\frac{2}{3}}(4x^3 + 2x) =$

$$\frac{2x(2x^2+1)}{3\left(x^4+x^2+1\right)^{\frac{2}{3}}}$$

The Power Rule

$$\left(\left(g\left(x\right)\right)^{n}\right)' = n\left(g\left(x\right)\right)^{n-1} g'\left(x\right)$$

The Power Rule

$$\left((g(x))^{n}\right)' = n\,(g(x))^{n-1}\,g'(x)$$

Example 4a: Find $f'(x)$ if $f(x) = \sqrt[3]{x^4 + x^2 + 1}$.

The Power Rule

$$\left((g(x))^n\right)' = n\,(g(x))^{n-1}\,g'(x)$$

Example 4a: Find $f'(x)$ if $f(x) = \sqrt[3]{x^4 + x^2 + 1}$.

We write $f(x) = (g(x))^{\frac{1}{3}}$ where $g(x) =$

The Power Rule

$$\left((g(x))^n\right)' = n\,(g(x))^{n-1}\,g'(x)$$

Example 4a: Find $f'(x)$ if $f(x) = \sqrt[3]{x^4 + x^2 + 1}$.

We write $f(x) = (g(x))^{\frac{1}{3}}$ where $g(x) = x^4 + x^2 + 1$.

The Power Rule

$$\left((g(x))^n\right)' = n\,(g(x))^{n-1}\,g'(x)$$

Example 4a: Find $f'(x)$ if $f(x) = \sqrt[3]{x^4 + x^2 + 1}$.

We write $f(x) = (g(x))^{\frac{1}{3}}$ where $g(x) = x^4 + x^2 + 1$.

Then

$f'(x) =$

The Power Rule

$$\left((g(x))^n\right)' = n\,(g(x))^{n-1}\,g'(x)$$

Example 4a: Find $f'(x)$ if $f(x) = \sqrt[3]{x^4 + x^2 + 1}$.

We write $f(x) = (g(x))^{\frac{1}{3}}$ where $g(x) = x^4 + x^2 + 1$.

Then

$f'(x) = \frac{1}{3}(g(x))^{-\frac{2}{3}}g'(x) =$

The Power Rule

$$\left((g(x))^n\right)' = n\,(g(x))^{n-1}\,g'(x)$$

Example 4a: Find $f'(x)$ if $f(x) = \sqrt[3]{x^4 + x^2 + 1}$.

We write $f(x) = (g(x))^{\frac{1}{3}}$ where $g(x) = x^4 + x^2 + 1$.

Then

$$f'(x) = \tfrac{1}{3}(g(x))^{-\frac{2}{3}}g'(x) = \tfrac{1}{3}\left(x^4 + x^2 + 1\right)^{-\frac{2}{3}}(4x^3 + 2x) =$$

The Power Rule

$$\left((g(x))^n\right)' = n\,(g(x))^{n-1}\,g'(x)$$

Example 4a: Find $f'(x)$ if $f(x) = \sqrt[3]{x^4 + x^2 + 1}$.

We write $f(x) = (g(x))^{\frac{1}{3}}$ where $g(x) = x^4 + x^2 + 1$.

Then

$$f'(x) = \frac{1}{3}(g(x))^{-\frac{2}{3}}g'(x) = \frac{1}{3}\left(x^4 + x^2 + 1\right)^{-\frac{2}{3}}\left(4x^3 + 2x\right) =$$

The Power Rule

$$\left((g(x))^n\right)' = n\,(g(x))^{n-1}\,g'(x)$$

Example 4a: Find $f'(x)$ if $f(x) = \sqrt[3]{x^4 + x^2 + 1}$.

We write $f(x) = (g(x))^{\frac{1}{3}}$ where $g(x) = x^4 + x^2 + 1$.

Then

$$f'(x) = \frac{1}{3}(g(x))^{-\frac{2}{3}}g'(x) = \frac{1}{3}\left(x^4 + x^2 + 1\right)^{-\frac{2}{3}}(4x^3 + 2x) =$$

$$\frac{2x(2x^2+1)}{3\left(x^4 + x^2 + 1\right)^{\frac{2}{3}}}$$

Example 5: Find $f'(x)$ if $f(x) = \left(\dfrac{4x - 3}{2x + 1}\right)^8$.

Example 5: Find $f'(x)$ if $f(x) = \left(\frac{4x-3}{2x+1}\right)^8$.

We have $f'(x) =$

Example 5: Find $f'(x)$ if $f(x) = \left(\frac{4x-3}{2x+1}\right)^8$.

We have $f'(x) = 8\left(\frac{4x-3}{2x+1}\right)^7 \left(\frac{4x-3}{2x+1}\right)' =$

Example 5: Find $f'(x)$ if $f(x) = \left(\dfrac{4x-3}{2x+1}\right)^8$.

We have $f'(x) = 8\left(\dfrac{4x-3}{2x+1}\right)^7 \left(\dfrac{4x-3}{2x+1}\right)' =$

$$8\left(\frac{4x-3}{2x+1}\right)^7 \left(\frac{(2x+1)(4x-3)' - (4x-3)(2x+1)'}{(2x+1)^2}\right) =$$

Example 5: Find $f'(x)$ if $f(x) = \left(\frac{4x-3}{2x+1}\right)^8$.

We have $f'(x) = 8\left(\frac{4x-3}{2x+1}\right)^7 \left(\frac{4x-3}{2x+1}\right)' =$

$$8\left(\frac{4x-3}{2x+1}\right)^7 \left(\frac{(2x+1)(4x-3)' - (4x-3)(2x+1)'}{(2x+1)^2}\right) =$$

$$8\left(\frac{4x-3}{2x+1}\right)^7 \left(\frac{(2x+1)4 - (4x-3)2}{(2x+1)^2}\right) =$$

Example 5: Find $f'(x)$ if $f(x) = \left(\frac{4x-3}{2x+1}\right)^8$.

We have $f'(x) = 8\left(\frac{4x-3}{2x+1}\right)^7\left(\frac{4x-3}{2x+1}\right)' =$

$$8\left(\frac{4x-3}{2x+1}\right)^7\left(\frac{(2x+1)(4x-3)'-(4x-3)(2x+1)'}{(2x+1)^2}\right) =$$

$$8\left(\frac{4x-3}{2x+1}\right)^7\left(\frac{(2x+1)4-(4x-3)2}{(2x+1)^2}\right) =$$

$$8\left(\frac{4x-3}{2x+1}\right)^7\left(\frac{8x+4-8x+6}{(2x+1)^2}\right) =$$

Example 5: Find $f'(x)$ if $f(x) = \left(\frac{4x-3}{2x+1}\right)^8$.

We have $f'(x) = 8\left(\frac{4x-3}{2x+1}\right)^7 \left(\frac{4x-3}{2x+1}\right)' =$

$$8\left(\frac{4x-3}{2x+1}\right)^7 \left(\frac{(2x+1)(4x-3)' - (4x-3)(2x+1)'}{(2x+1)^2}\right) =$$

$$8\left(\frac{4x-3}{2x+1}\right)^7 \left(\frac{(2x+1)4 - (4x-3)2}{(2x+1)^2}\right) =$$

$$8\left(\frac{4x-3}{2x+1}\right)^7 \left(\frac{8x+4-8x+6}{(2x+1)^2}\right) =$$

$$8\left(\frac{4x-3}{2x+1}\right)^7 \left(\frac{10}{(2x+1)^2}\right) =$$

Example 5: Find $f'(x)$ if $f(x) = \left(\frac{4x-3}{2x+1}\right)^8$.

We have $f'(x) = 8\left(\frac{4x-3}{2x+1}\right)^7 \left(\frac{4x-3}{2x+1}\right)' =$

$$8\left(\frac{4x-3}{2x+1}\right)^7 \left(\frac{(2x+1)(4x-3)' - (4x-3)(2x+1)'}{(2x+1)^2}\right) =$$

$$8\left(\frac{4x-3}{2x+1}\right)^7 \left(\frac{(2x+1)4 - (4x-3)2}{(2x+1)^2}\right) =$$

$$8\left(\frac{4x-3}{2x+1}\right)^7 \left(\frac{8x+4-8x+6}{(2x+1)^2}\right) =$$

$$8\left(\frac{4x-3}{2x+1}\right)^7 \left(\frac{10}{(2x+1)^2}\right) =$$

Example 5: Find $f'(x)$ if $f(x) = \left(\frac{4x-3}{2x+1}\right)^8$.

We have $f'(x) = 8\left(\frac{4x-3}{2x+1}\right)^7\left(\frac{4x-3}{2x+1}\right)' =$

$$8\left(\frac{4x-3}{2x+1}\right)^7\left(\frac{(2x+1)(4x-3)' - (4x-3)(2x+1)'}{(2x+1)^2}\right) =$$

$$8\left(\frac{4x-3}{2x+1}\right)^7\left(\frac{(2x+1)4 - (4x-3)2}{(2x+1)^2}\right) =$$

$$8\left(\frac{4x-3}{2x+1}\right)^7\left(\frac{8x+4-8x+6}{(2x+1)^2}\right) =$$

$$8\left(\frac{4x-3}{2x+1}\right)^7\left(\frac{10}{(2x+1)^2}\right) = \quad 80\frac{(4x-3)^7}{(2x+1)^9}$$

Example 6: Find $f'(x)$ if $f(x) = \dfrac{(4x-3)^8}{(2x+1)^5}$.

Example 6: Find $f'(x)$ if $f(x) = \dfrac{(4x-3)^8}{(2x+1)^5}$.

First we use the Quotient Rule:

Example 6: Find $f'(x)$ if $f(x) = \dfrac{(4x-3)^8}{(2x+1)^5}$.

First we use the Quotient Rule:

$f'(x) =$

Example 6: Find $f'(x)$ if $f(x) = \dfrac{(4x-3)^8}{(2x+1)^5}$.

First we use the Quotient Rule:

$$f'(x) = \frac{[(2x+1)^5]\,[(4x-3)^8]' - (4x-3)^8\,[(2x+1)^5]'}{[(2x+1)^5]^2}$$

Example 6: Find $f'(x)$ if $f(x) = \dfrac{(4x-3)^8}{(2x+1)^5}$.

First we use the Quotient Rule:

$$f'(x) = \frac{[(2x+1)^5]\,[(4x-3)^8]' - (4x-3)^8\,[(2x+1)^5]'}{[(2x+1)^5]^2}$$

and then the Power Rule:

Example 6: Find $f'(x)$ if $f(x) = \dfrac{(4x-3)^8}{(2x+1)^5}$.

First we use the Quotient Rule:

$$f'(x) = \frac{\left[(2x+1)^5\right]\left[(4x-3)^8\right]' - (4x-3)^8\left[(2x+1)^5\right]'}{\left[(2x+1)^5\right]^2}$$

and then the Power Rule:

$$f'(x) =$$

Example 6: Find $f'(x)$ if $f(x) = \dfrac{(4x-3)^8}{(2x+1)^5}$.

First we use the Quotient Rule:

$$f'(x) = \frac{[(2x+1)^5]\,[(4x-3)^8]' - (4x-3)^8\,[(2x+1)^5]'}{[(2x+1)^5]^2}$$

and then the Power Rule:

$$f'(x) =$$

$$\frac{[(2x+1)^5]\,8(4x-3)^{8-1}(4x-3)' - (4x-3)^8 5(2x+1)^{5-1}(2x+1)'}{(2x+1)^{5\times 2}} =$$

Example 6: Find $f'(x)$ if $f(x) = \dfrac{(4x-3)^8}{(2x+1)^5}$.

First we use the Quotient Rule:

$$f'(x) = \frac{\left[(2x+1)^5\right]\left[(4x-3)^8\right]' - (4x-3)^8\left[(2x+1)^5\right]'}{\left[(2x+1)^5\right]^2}$$

and then the Power Rule:

$$f'(x) =$$

$$\frac{\left[(2x+1)^5\right]8(4x-3)^{8-1}(4x-3)' - (4x-3)^8 5(2x+1)^{5-1}(2x+1)'}{(2x+1)^{5\times 2}} =$$

$$\frac{(2x+1)^5 8(4x-3)^7(4) - (4x-3)^8 5(2x+1)^4(2)}{(2x+1)^{10}} =$$

$$\frac{32(2x+1)^5(4x-3)^7-10(4x-3)^8(2x+1)^4}{(2x+1)^{10}} =$$

$$\frac{32(2x+1)^5(4x-3)^7 - 10(4x-3)^8(2x+1)^4}{(2x+1)^{10}} =$$

$$\frac{32(2x+1)(4x-3)^7 - 10(4x-3)^8}{(2x+1)^6} =$$

$$\frac{32(2x+1)^5(4x-3)^7 - 10(4x-3)^8(2x+1)^4}{(2x+1)^{10}} =$$

$$\frac{32(2x+1)(4x-3)^7 - 10(4x-3)^8}{(2x+1)^6} =$$

$$(4x-3)^7\,\frac{32(2x+1) - 10(4x-3)}{(2x+1)^6} =$$

$$\frac{32(2x+1)^5(4x-3)^7 - 10(4x-3)^8(2x+1)^4}{(2x+1)^{10}} =$$

$$\frac{32(2x+1)(4x-3)^7 - 10(4x-3)^8}{(2x+1)^6} =$$

$$(4x-3)^7\,\frac{32(2x+1) - 10(4x-3)}{(2x+1)^6} =$$

$$\frac{(4x-3)^7}{(2x+1)^6}\,[64x + 32 - 40x + 30] =$$

$$\frac{32(2x+1)^5(4x-3)^7 - 10(4x-3)^8(2x+1)^4}{(2x+1)^{10}} =$$

$$\frac{32(2x+1)(4x-3)^7 - 10(4x-3)^8}{(2x+1)^6} =$$

$$(4x-3)^7\,\frac{32(2x+1) - 10(4x-3)}{(2x+1)^6} =$$

$$\frac{(4x-3)^7}{(2x+1)^6}\,[64x+32-40x+30] = \frac{(4x-3)^7}{(2x+1)^6}\,[24x+62] =$$

$$\frac{32(2x+1)^5(4x-3)^7 - 10(4x-3)^8(2x+1)^4}{(2x+1)^{10}} =$$

$$\frac{32(2x+1)(4x-3)^7 - 10(4x-3)^8}{(2x+1)^6} =$$

$$(4x-3)^7\,\frac{32(2x+1) - 10(4x-3)}{(2x+1)^6} =$$

$$\frac{(4x-3)^7}{(2x+1)^6}\,[64x+32-40x+30] = \frac{(4x-3)^7}{(2x+1)^6}\,[24x+62] =$$

$$\frac{32(2x+1)^5(4x-3)^7 - 10(4x-3)^8(2x+1)^4}{(2x+1)^{10}} =$$

$$\frac{32(2x+1)(4x-3)^7 - 10(4x-3)^8}{(2x+1)^6} =$$

$$(4x-3)^7\,\frac{32(2x+1) - 10(4x-3)}{(2x+1)^6} =$$

$$\frac{(4x-3)^7}{(2x+1)^6}[64x+32-40x+30] = \frac{(4x-3)^7}{(2x+1)^6}[24x+62] =$$

$$2\frac{(4x-3)^7}{(2x+1)^6}[12x+31]$$