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These rules are sufficient for the differentiation of all polynomials.

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$$(f(x)g(x))' = f(x)g'(x) + f'(x)g(x)$$

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Notice that  $f'(x)g'(x) = 1(1) = 1$ , which is quite different, and incorrect.

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