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Taking limits,

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Remember: $\frac{\frac{a}{b}}{\frac{c}{d}} = \left(\frac{a}{b}\right) \left(\frac{d}{c}\right)$

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