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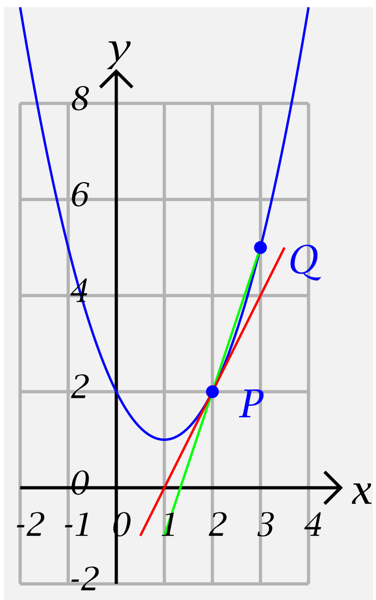

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We can rewrite the limit calculation, using $h = b - a$ and $b = a + h$, in the form

$$\lim_{Q \to P} m_{PQ} = \lim_{b \to a} \frac{f(b) - f(a)}{b - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{(a+h) - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Example 1: $f(x) = x^2$, $P = (2, 4)$.

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Alternative limit formulation:

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$$\lim_{h \to 0} \frac{2^2 + 2(2)h + h^2 - 2^2}{h} =$$

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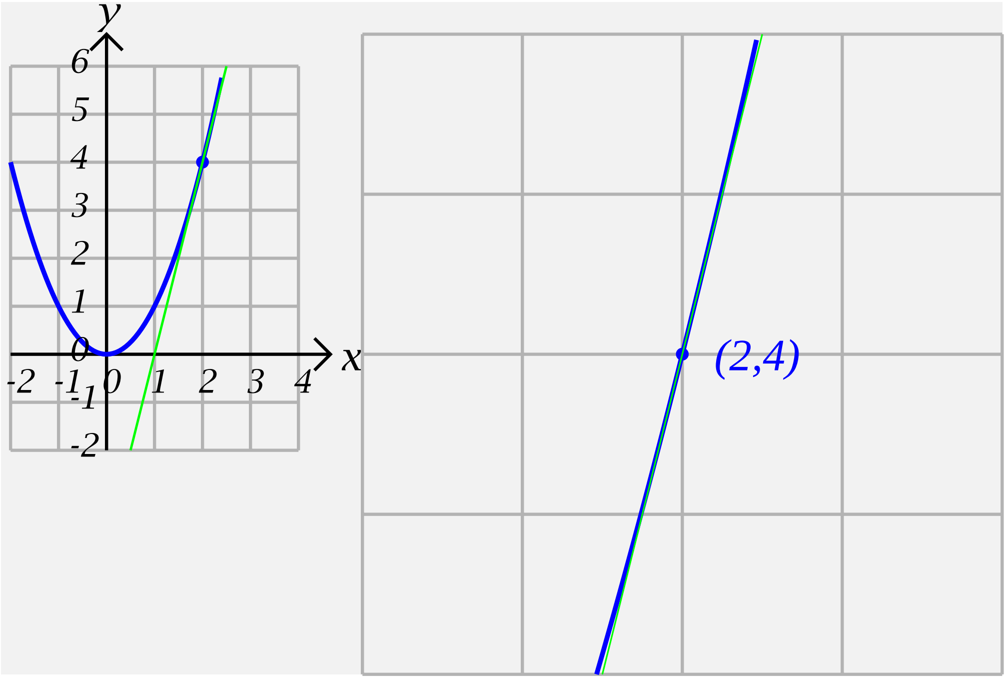

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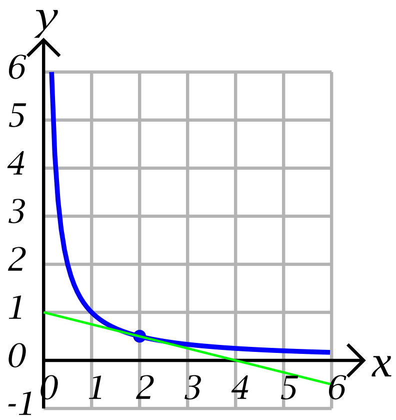

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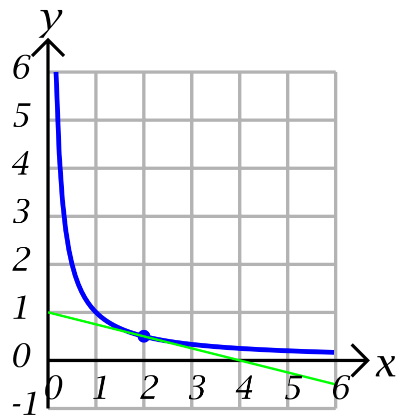

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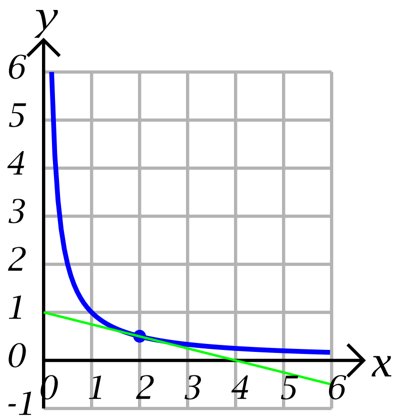

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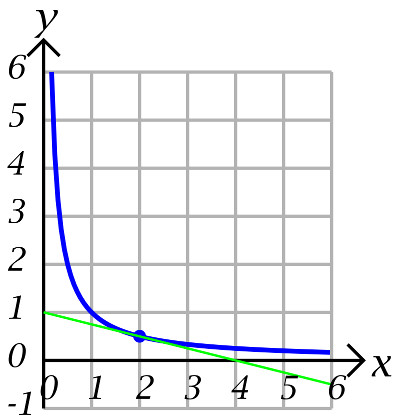

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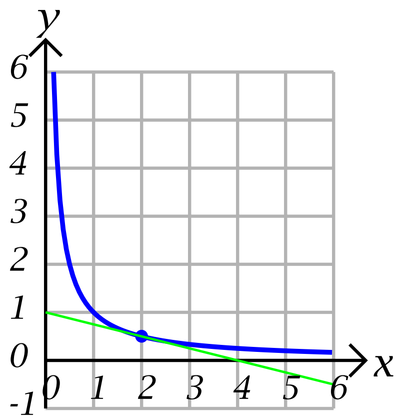

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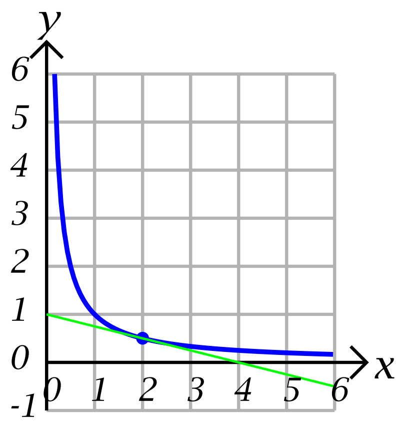

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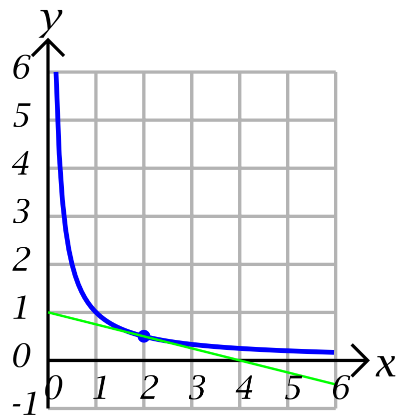

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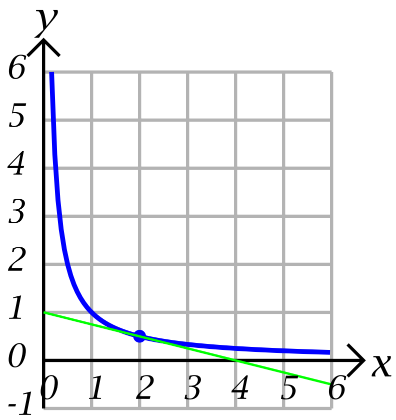

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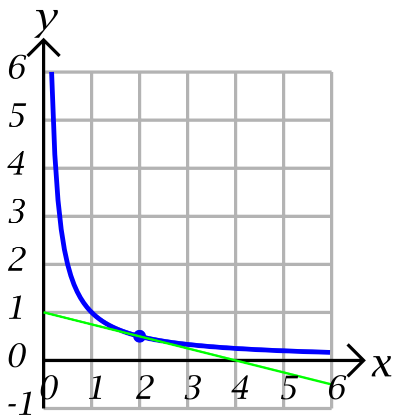

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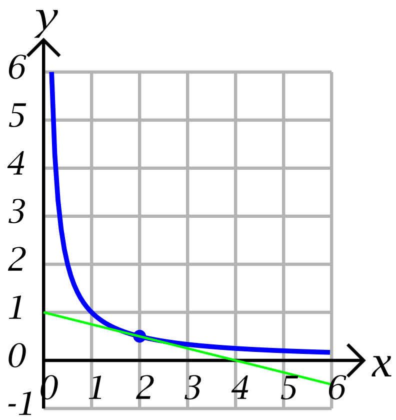

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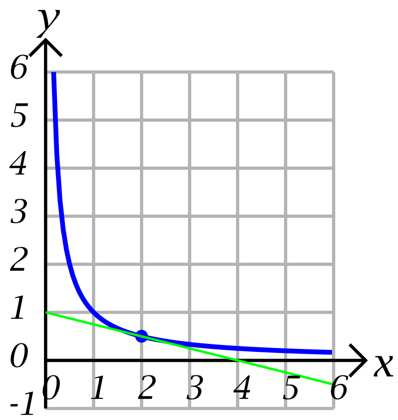

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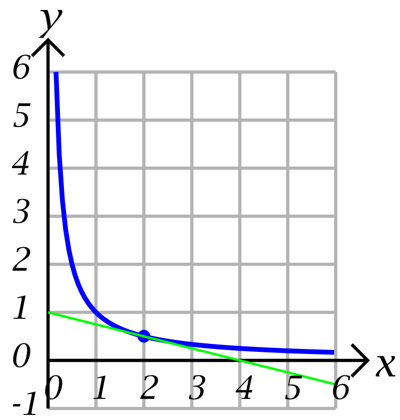

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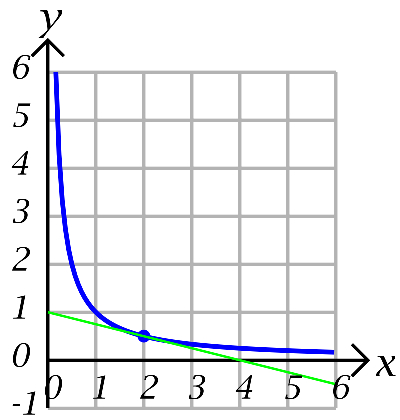

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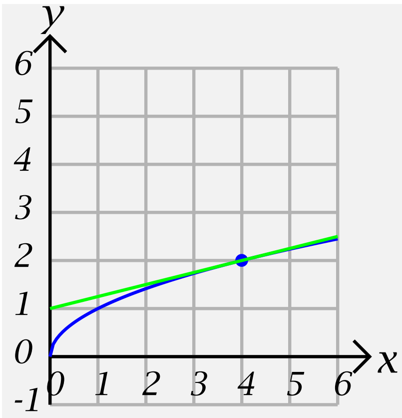

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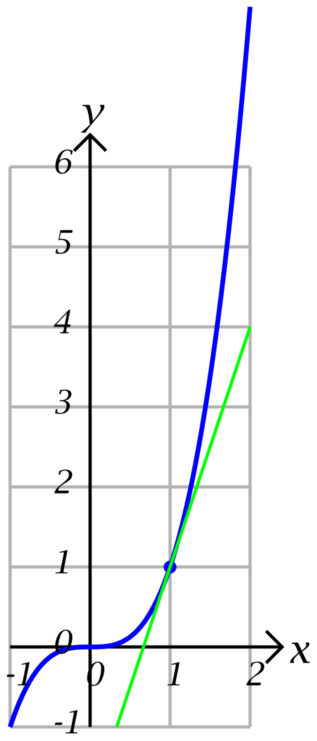

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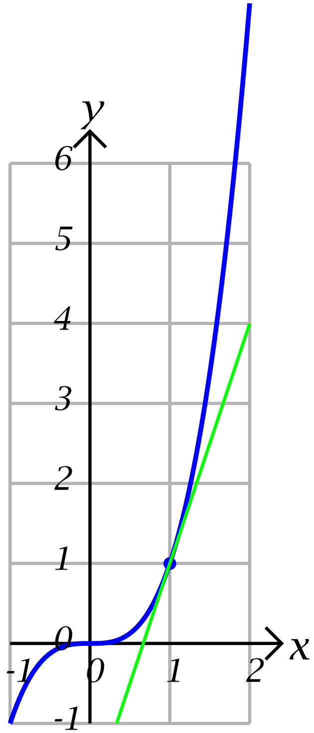

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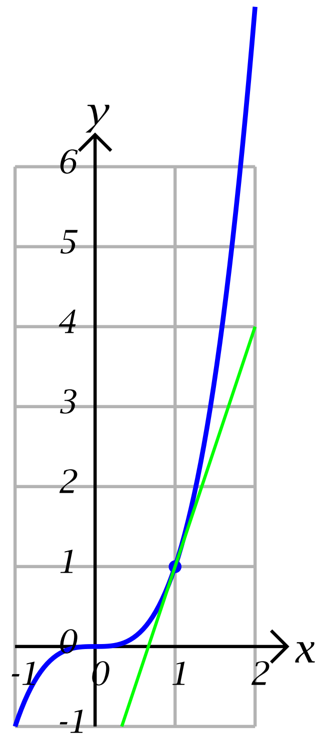

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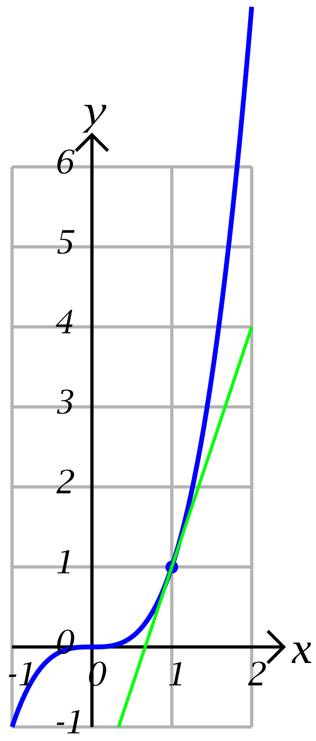

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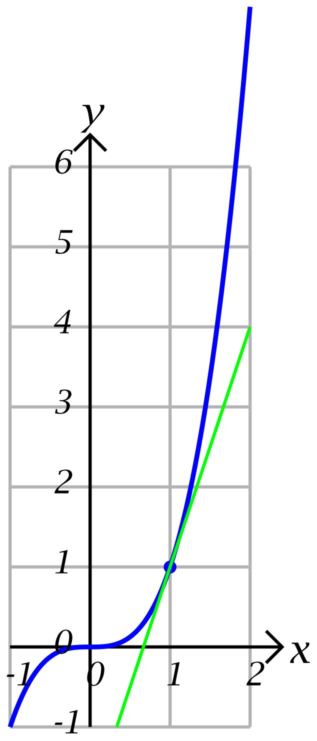

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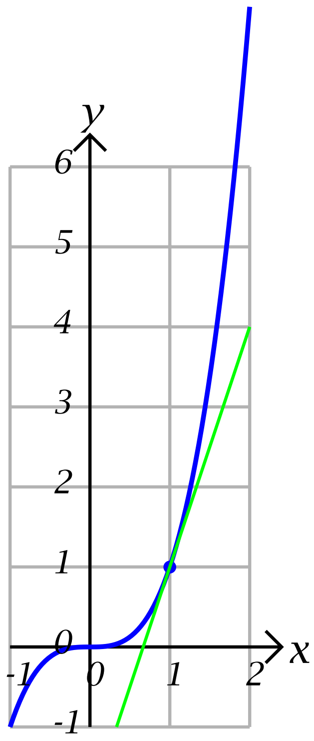

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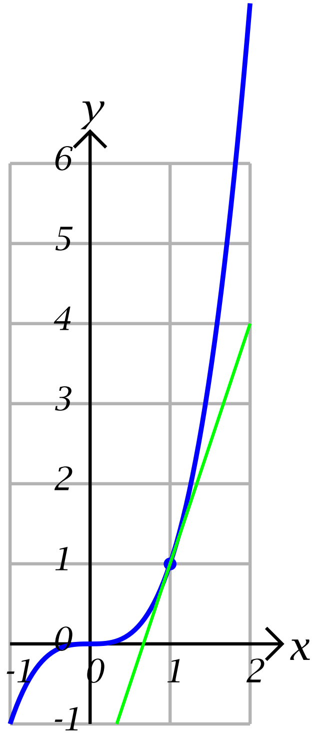

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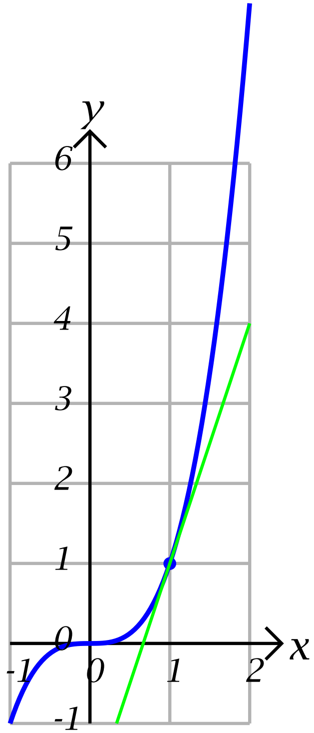

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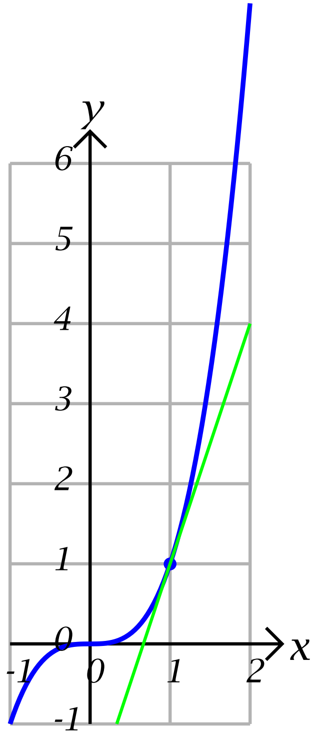

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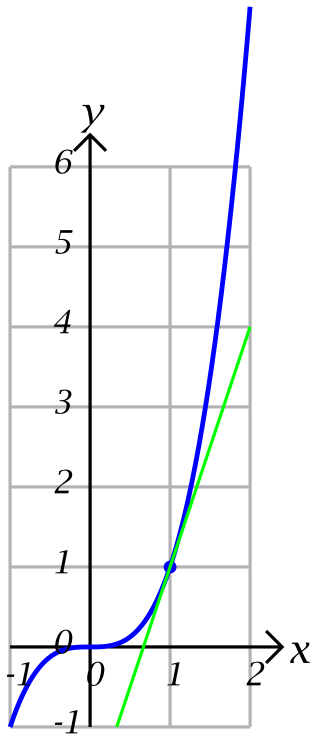

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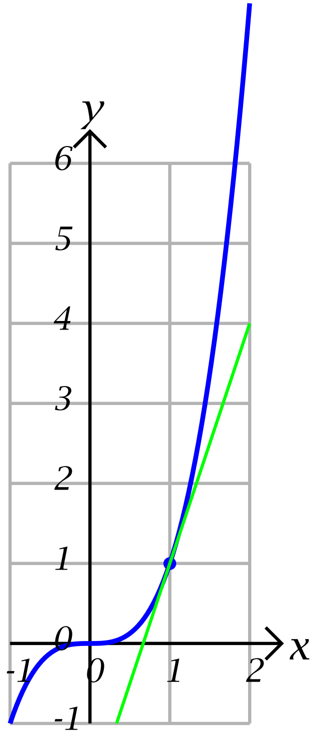

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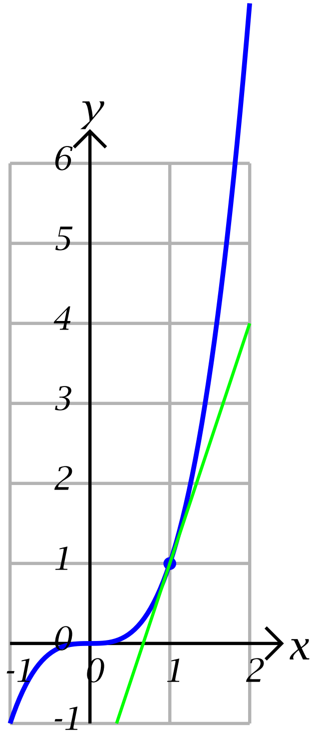

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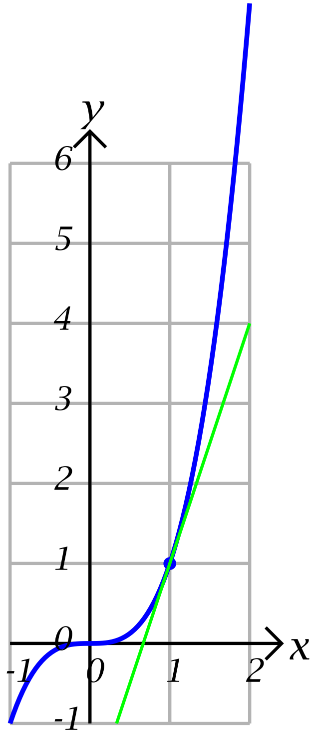
$$\frac{(b-1)(b^2 + b + 1)}{b - 1} = b^2 + b + 1, \text{ so}$$

$$\lim_{Q \to P} m_{PQ} = \lim_{b \to 1} = b^2 + b + 1 = 3 \text{ exists,}$$

so the equation of the tangent line is

$y - 1 = 3(x - 1)$



Using the alternative limit formulation, we have

$$m = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{(1+h)^3 - 1^3}{h} =$$

$$\lim_{h \to 0} \frac{1^3 + 3(1)^2 h + 3(1)h^2 + h^3 - 1^3}{h} = \lim_{h \to 0} \frac{3h + 3h^2 + h^3}{h} =$$

$$\lim_{h \to 0} \frac{h(3 + 3h + h^2)}{h} = \lim_{h \to 0}(3 + 3h + h^2) = (3 + 3(0) + 0^2) =$$

Example 5: $f(x) = x^3$, $P = (1, 1)$.

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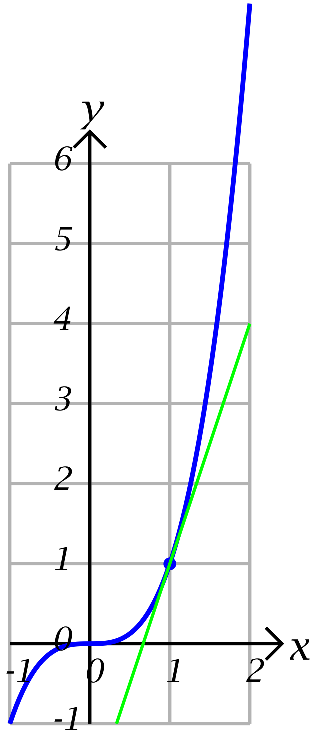

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$$\frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

Example: $s(t) = 2t^2 - 6t + 3$

Time Interval = $[t_1, t_2]$	Average Velocity = $\frac{s(t_2)-s(t_1)}{t_2-t_1}$

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$[0, 1]$	$\dfrac{s(1)-s(0)}{1-0} = \dfrac{-1-3}{1} = -4$

Time Interval = $[t_1, t_2]$	Average Velocity = $\frac{s(t_2)-s(t_1)}{t_2-t_1}$
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$[0,3]$	$\dfrac{s(3)-s(0)}{3-0} = \dfrac{3-3}{3} = 0$

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On the general interval $[a, b]$ the Average Velocity is

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$$\frac{2(b-a)(b+a)-6(b-a)}{b-a} = 2(b+a)-6$$

Let us look at the Average Velocity near $t_1 = 3$:

Time Interval = $[3, t_2]$	Average Velocity = $2(3 + t_2) - 6$

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Time Interval = $[3, t_2]$	Average Velocity = $2(3 + t_2) - 6$
$[3, 4]$	$2(3 + 4) - 6 = 8$
$[3, 3.1]$	$2(3 + 3.1) - 6 = 6.2$

Time Interval = $[3, t_2]$	Average Velocity = $2(3 + t_2) - 6$
$[3, 4]$	$2(3 + 4) - 6 = 8$
$[3, 3.1]$	$2(3 + 3.1) - 6 = 6.2$
$[3, 3.01]$	$2(3 + 3.01) - 6 = 6.02$

Time Interval = $[3, t_2]$	Average Velocity = $2(3 + t_2) - 6$
$[3, 4]$	$2(3 + 4) - 6 = 8$
$[3, 3.1]$	$2(3 + 3.1) - 6 = 6.2$
$[3, 3.01]$	$2(3 + 3.01) - 6 = 6.02$
$[3, 3.001]$	$2(3 + 3.001) - 6 = 6.002$

Time Interval = $[3, t_2]$	Average Velocity = $2(3 + t_2) - 6$
$[3, 4]$	$2(3 + 4) - 6 = 8$
$[3, 3.1]$	$2(3 + 3.1) - 6 = 6.2$
$[3, 3.01]$	$2(3 + 3.01) - 6 = 6.02$
$[3, 3.001]$	$2(3 + 3.001) - 6 = 6.002$
$[3, 3.0001]$	$2(3 + 3.0001) - 6 = 6.0002$

Time Interval = $[3, t_2]$	Average Velocity = $2(3 + t_2) - 6$
$[3, 4]$	$2(3 + 4) - 6 = 8$
$[3, 3.1]$	$2(3 + 3.1) - 6 = 6.2$
$[3, 3.01]$	$2(3 + 3.01) - 6 = 6.02$
$[3, 3.001]$	$2(3 + 3.001) - 6 = 6.002$
$[3, 3.0001]$	$2(3 + 3.0001) - 6 = 6.0002$
$[3, 3.00001]$	$2(3 + 3.00001) - 6 = 6.00002$

and

Time Interval = $[t_1, 3]$	Average Velocity = $2(t_1 + 3) - 6$

Time Interval = $[t_1, 3]$	Average Velocity = $2(t_1 + 3) - 6$
$[2, 3]$	$2(2 + 3) - 6 = 4$

Time Interval = $[t_1, 3]$	Average Velocity = $2(t_1 + 3) - 6$
$[2, 3]$	$2(2 + 3) - 6 = 4$
$[2.9, 3]$	$2(2.9 + 3) - 6 = 5.8$

Time Interval = $[t_1, 3]$	Average Velocity = $2(t_1 + 3) - 6$
$[2, 3]$	$2(2 + 3) - 6 = 4$
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$[2.99, 3]$	$2(2.99 + 3) - 6 = 5.98$

Time Interval = $[t_1, 3]$	Average Velocity = $2(t_1 + 3) - 6$
$[2, 3]$	$2(2 + 3) - 6 = 4$
$[2.9, 3]$	$2(2.9 + 3) - 6 = 5.8$
$[2.99, 3]$	$2(2.99 + 3) - 6 = 5.98$
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If we take the limit as t approaches 3, we get:

$$\lim_{t \to 3} \frac{s(t) - s(3)}{t - 3} =$$

Time Interval = $[t_1, 3]$	Average Velocity = $2(t_1 + 3) - 6$
$[2, 3]$	$2(2 + 3) - 6 = 4$
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$$\lim_{t \to 3} \frac{s(t) - s(3)}{t - 3} = \lim_{t \to 3} 2(t + 3) - 6 =$$

Time Interval = $[t_1, 3]$	Average Velocity = $2(t_1 + 3) - 6$
$[2, 3]$	$2(2 + 3) - 6 = 4$
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If we take the limit as t approaches 3, we get:

$$\lim_{t \to 3} \frac{s(t) - s(3)}{t - 3} = \lim_{t \to 3} 2(t + 3) - 6 = 2(3 + 3) - 6 =$$

Time Interval = $[t_1, 3]$	Average Velocity = $2(t_1 + 3) - 6$
$[2, 3]$	$2(2 + 3) - 6 = 4$
$[2.9, 3]$	$2(2.9 + 3) - 6 = 5.8$
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If we take the limit as t approaches 3, we get:

$$\lim_{t \to 3} \frac{s(t) - s(3)}{t - 3} = \lim_{t \to 3} 2(t + 3) - 6 = 2(3 + 3) - 6 = 12 - 6 =$$

Time Interval = $[t_1, 3]$	Average Velocity = $2(t_1 + 3) - 6$
$[2, 3]$	$2(2 + 3) - 6 = 4$
$[2.9, 3]$	$2(2.9 + 3) - 6 = 5.8$
$[2.99, 3]$	$2(2.99 + 3) - 6 = 5.98$
$[2.999, 3]$	$2(2.999 + 3) - 6 = 5.998$
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$$\lim_{t \to 3} \frac{s(t) - s(3)}{t - 3} = \lim_{t \to 3} 2(t + 3) - 6 = 2(3 + 3) - 6 = 12 - 6 = 6.$$

This limit is called the

Time Interval = $[t_1, 3]$	Average Velocity = $2(t_1 + 3) - 6$
$[2, 3]$	$2(2 + 3) - 6 = 4$
$[2.9, 3]$	$2(2.9 + 3) - 6 = 5.8$
$[2.99, 3]$	$2(2.99 + 3) - 6 = 5.98$
$[2.999, 3]$	$2(2.999 + 3) - 6 = 5.998$
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This limit is called the Instantaneous Velocity at time $t = 3$.

In general, the Instantaneous Velocity at time $t = a$ is defined to be

Time Interval = $[t_1, 3]$	Average Velocity = $2(t_1 + 3) - 6$
$[2, 3]$	$2(2 + 3) - 6 = 4$
$[2.9, 3]$	$2(2.9 + 3) - 6 = 5.8$
$[2.99, 3]$	$2(2.99 + 3) - 6 = 5.98$
$[2.999, 3]$	$2(2.999 + 3) - 6 = 5.998$
$[2.9999, 3]$	$2(2.9999 + 3) - 6 = 5.9998$
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$$\lim_{t \to 3} \frac{s(t) - s(3)}{t - 3} = \lim_{t \to 3} 2(t + 3) - 6 = 2(3 + 3) - 6 = 12 - 6 = 6.$$

This limit is called the Instantaneous Velocity at time $t = 3$.

In general, the Instantaneous Velocity at time $t = a$ is defined to be

Time Interval = $[t_1, 3]$	Average Velocity = $2(t_1 + 3) - 6$
$[2, 3]$	$2(2 + 3) - 6 = 4$
$[2.9, 3]$	$2(2.9 + 3) - 6 = 5.8$
$[2.99, 3]$	$2(2.99 + 3) - 6 = 5.98$
$[2.999, 3]$	$2(2.999 + 3) - 6 = 5.998$
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If we take the limit as t approaches 3, we get:

$$\lim_{t\to 3} \frac{s(t) - s(3)}{t - 3} = \lim_{t\to 3} 2(t + 3) - 6 = 2(3 + 3) - 6 = 12 - 6 = 6.$$

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In general, the Instantaneous Velocity at time $t = a$ is defined to be

$$\lim_{t\to a} \frac{s(t) - s(a)}{t - a}$$

With our example function $s(t) = 2t^2 - 6t + 3$ we then have

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$$\lim_{t \to a} \frac{s(t) - s(a)}{t - a} = \lim_{t \to a} 2(a + t) - 6 = 2(a + a) - 6 = 4a - 6$$

Another equivalent and very useful formula for the instantaneous velocity at time t is:

With our example function $s(t) = 2t^2 - 6t + 3$ we then have

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