

## Exercises for Differentiation Techniques

Use the rules of differentiation to find  $f(x)'$ :

**(1)**  $f(x) = (x + 1)^2$

Solution

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**(2)**  $f(x) = (x + 2)^3$

Solution

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**(3)**  $f(x) = (x + 3)^4$

Solution

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**(4)**  $f(x) = x + \sqrt{x}$

Solution

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**(5)**  $f(x) = \frac{1}{x+1}$

Solution

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**(6)**  $f(x) = \frac{\sqrt{x}}{x+5}$

Solution

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**(7)**  $f(x) = \frac{x}{x+1}$

Solution

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**(8)**  $f(x) = \frac{x}{\sqrt{x}+1}$

Solution

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## Solutions

(1)

$$f(x) = (x + 1)^2 = x^2 + 2x^1 + 1$$

$$f'(x) =$$

[Back to Questions](#)

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(1)

$$f(x) = (x + 1)^2 = x^2 + 2x^1 + 1$$

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[Back to Questions](#)

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$$f(x) = (x + 1)^2 = x^2 + 2x^1 + 1$$

$$f'(x) = 2x^{2-1} + 2x^{1-1} + 0 = 2x + 2 =$$

[Back to Questions](#)

## Solutions

(1)

$$f(x) = (x + 1)^2 = x^2 + 2x^1 + 1$$

[Back to Questions](#)

$$f'(x) = 2x^{2-1} + 2x^{1-1} + 0 = 2x + 2 = 2(x + 1)$$

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(2)

$$f(x) = (x + 2)^3 = x^3 + 3x^2 + 3x^1 + 1$$

$$f'(x) =$$

[Back to Questions](#)

(2)

$$f(x) = (x + 2)^3 = x^3 + 3x^2 + 3x^1 + 1$$

$$f'(x) = 3x^{3-1} + 3(2)x^{2-1} + 3x^{1-1} + 0 =$$

[Back to Questions](#)

(2)

$$f(x) = (x + 2)^3 = x^3 + 3x^2 + 3x^1 + 1$$

$$f'(x) = 3x^{3-1} + 3(2)x^{2-1} + 3x^{1-1} + 0 = 3x^2 + 6x + 3 =$$

[Back to Questions](#)

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$$f(x) = (x + 2)^3 = x^3 + 3x^2 + 3x^1 + 1$$

$$f'(x) = 3x^{3-1} + 3(2)x^{2-1} + 3x^{1-1} + 0 = 3x^2 + 6x + 3 = 3(x^2 + 2x + 1) =$$

[Back to Questions](#)

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$$f(x) = (x + 2)^3 = x^3 + 3x^2 + 3x^1 + 1$$

$$f'(x) = 3x^{3-1} + 3(2)x^{2-1} + 3x^{1-1} + 0 = 3x^2 + 6x + 3 = 3(x^2 + 2x + 1) = 3(x + 2)^2$$

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[Back to Questions](#)

$$(3) \quad f(x) = (x + 3)^4 = x^4 + 4x^3(3) + 6x^23^3 + 4x^13^2 + 3^4 = x^4 + 12x^3 + 54x^2 + 108x^1 + 81$$

$$f'(x) =$$

[Back to Questions](#)

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[Back to Questions](#)

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[Back to Questions](#)

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$$f'(x) = 4x^{4-1} + 12(3)x^{3-1} + 54(2)x^{2-1} + 108x^{1-1} + 0 = 4x^3 + 36x^2 + 108x + 108 = 4(x^3 + 9x^2 + 27x + 27) =$$

[Back to Questions](#)

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[Back to Questions](#)

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$$4(x^3 + 3x^2 \cdot 3 + 3x \cdot 3^2 + 3^3) = 4(x + 3)^3$$

[Back to Questions](#)

(4)

$$f(x) = x + \sqrt{x} = x + x^{\frac{1}{2}}.$$

$$f'(x) =$$

[Back to Questions](#)

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$$f(x) = x + \sqrt{x} = x + x^{\frac{1}{2}}.$$

$$f'(x) = 1 + \frac{1}{2}x^{\frac{1}{2}-1} =$$

[Back to Questions](#)

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$$f(x) = x + \sqrt{x} = x + x^{\frac{1}{2}}.$$

$$f'(x) = 1 + \frac{1}{2}x^{\frac{1}{2}-1} = 1 + \frac{1}{2}x^{-\frac{1}{2}} =$$

[Back to Questions](#)

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$$f(x) = x + \sqrt{x} = x + x^{\frac{1}{2}}.$$

$$f'(x) = 1 + \frac{1}{2}x^{\frac{1}{2}-1} = 1 + \frac{1}{2}x^{-\frac{1}{2}} = 1 + \frac{1}{2\sqrt{x}}$$

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[Back to Questions](#)

(5)

$f(x) = \frac{1}{x+1}$ . Using the Quotient Rule:

$f'(x) =$

[Back to Questions](#)

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$f(x) = \frac{1}{x+1}$ . Using the Quotient Rule:

$$f'(x) = \frac{(x+1)(1)' - 1(x+1)'}{(x+1)^2} =$$

[Back to Questions](#)

(5)

$f(x) = \frac{1}{x+1}$ . Using the Quotient Rule:

$$f'(x) = \frac{(x+1)(1)' - 1(x+1)'}{(x+1)^2} = \frac{(x+1)(0) - 1(x'+1)'}{(x+1)^2} =$$

[Back to Questions](#)

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$f(x) = \frac{1}{x+1}$ . Using the Quotient Rule:

$$f'(x) = \frac{(x+1)(1)' - 1(x+1)'}{(x+1)^2} = \frac{(x+1)(0) - 1(x'+1')}{(x+1)^2} = \frac{0 - 1(1+0)}{(x+1)^2} =$$

[Back to Questions](#)

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$f(x) = \frac{1}{x+1}$ . Using the Quotient Rule:

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[Back to Questions](#)

**(6)**

$$f(x) = \frac{\sqrt{x}}{x+5} = \frac{x^{\frac{1}{2}}}{x+5}.$$

$$f'(x) =$$

[Back to Questions](#)

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$$f(x) = \frac{\sqrt{x}}{x+5} = \frac{x^{\frac{1}{2}}}{x+5}.$$

$$f'(x) = \frac{(x+5)(x^{\frac{1}{2}})' - x^{\frac{1}{2}}(x+5)'}{(x+5)^2} =$$

[Back to Questions](#)

**(6)**

$$f(x) = \frac{\sqrt{x}}{x+5} = \frac{x^{\frac{1}{2}}}{x+5}.$$

$$f'(x) = \frac{(x+5)(x^{\frac{1}{2}})' - x^{\frac{1}{2}}(x+5)'}{(x+5)^2} = \frac{(x+5)\left(\frac{1}{2}x^{\frac{1}{2}-1}\right) - x^{\frac{1}{2}}(x'+5')}{(x+5)^2} =$$

[Back to Questions](#)

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$$f(x) = \frac{\sqrt{x}}{x+5} = \frac{x^{\frac{1}{2}}}{x+5}.$$

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$$\frac{(x+5)\left(\frac{1}{2}x^{-\frac{1}{2}}\right) - x^{\frac{1}{2}}(1+0)}{(x+5)^2} =$$

[Back to Questions](#)

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$$f(x) = \frac{\sqrt{x}}{x+5} = \frac{x^{\frac{1}{2}}}{x+5}.$$

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$$\frac{(x+5)\left(\frac{1}{2}x^{-\frac{1}{2}}\right) - x^{\frac{1}{2}}(1+0)}{(x+5)^2} = \frac{x\frac{1}{2}x^{-\frac{1}{2}} + 5\frac{1}{2}x^{-\frac{1}{2}} - x^{\frac{1}{2}}}{(x+5)^2} =$$

[Back to Questions](#)

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$$f(x) = \frac{\sqrt{x}}{x+5} = \frac{x^{\frac{1}{2}}}{x+5}.$$

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$$\frac{(x+5)\left(\frac{1}{2}x^{-\frac{1}{2}}\right) - x^{\frac{1}{2}}(1+0)}{(x+5)^2} = \frac{x\frac{1}{2}x^{-\frac{1}{2}} + 5\frac{1}{2}x^{-\frac{1}{2}} - x^{\frac{1}{2}}}{(x+5)^2} = \frac{\frac{1}{2}x^{\frac{1}{2}} + \frac{5}{2\sqrt{x}} - x^{\frac{1}{2}}}{(x+5)^2} =$$

[Back to Questions](#)

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$$f(x) = \frac{\sqrt{x}}{x+5} = \frac{x^{\frac{1}{2}}}{x+5}.$$

$$f'(x) = \frac{(x+5)(x^{\frac{1}{2}})' - x^{\frac{1}{2}}(x+5)'}{(x+5)^2} = \frac{(x+5)\left(\frac{1}{2}x^{\frac{1}{2}-1}\right) - x^{\frac{1}{2}}(x'+5')}{(x+5)^2} =$$

$$\frac{(x+5)\left(\frac{1}{2}x^{-\frac{1}{2}}\right) - x^{\frac{1}{2}}(1+0)}{(x+5)^2} = \frac{x\frac{1}{2}x^{-\frac{1}{2}} + 5\frac{1}{2}x^{-\frac{1}{2}} - x^{\frac{1}{2}}}{(x+5)^2} = \frac{\frac{1}{2}x^{\frac{1}{2}} + \frac{5}{2\sqrt{x}} - x^{\frac{1}{2}}}{(x+5)^2} =$$

$$\frac{\frac{5}{2\sqrt{x}} - \frac{1}{2}x^{\frac{1}{2}}}{(x+5)^2} =$$

[Back to Questions](#)

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$$f(x) = \frac{\sqrt{x}}{x+5} = \frac{x^{\frac{1}{2}}}{x+5}.$$

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$$\frac{(x+5)\left(\frac{1}{2}x^{-\frac{1}{2}}\right) - x^{\frac{1}{2}}(1+0)}{(x+5)^2} = \frac{x\frac{1}{2}x^{-\frac{1}{2}} + 5\frac{1}{2}x^{-\frac{1}{2}} - x^{\frac{1}{2}}}{(x+5)^2} = \frac{\frac{1}{2}x^{\frac{1}{2}} + \frac{5}{2\sqrt{x}} - x^{\frac{1}{2}}}{(x+5)^2} =$$

$$\frac{\frac{5}{2\sqrt{x}} - \frac{1}{2}x^{\frac{1}{2}}}{(x+5)^2} = \frac{\frac{5}{\sqrt{x}} - \sqrt{x}}{2(x+5)^2} =$$

[Back to Questions](#)

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$$f(x) = \frac{\sqrt{x}}{x+5} = \frac{x^{\frac{1}{2}}}{x+5}.$$

$$f'(x) = \frac{(x+5)(x^{\frac{1}{2}})' - x^{\frac{1}{2}}(x+5)'}{(x+5)^2} = \frac{(x+5)\left(\frac{1}{2}x^{\frac{1}{2}-1}\right) - x^{\frac{1}{2}}(x'+5')}{(x+5)^2} =$$

$$\frac{(x+5)\left(\frac{1}{2}x^{-\frac{1}{2}}\right) - x^{\frac{1}{2}}(1+0)}{(x+5)^2} = \frac{x\frac{1}{2}x^{-\frac{1}{2}} + 5\frac{1}{2}x^{-\frac{1}{2}} - x^{\frac{1}{2}}}{(x+5)^2} = \frac{\frac{1}{2}x^{\frac{1}{2}} + \frac{5}{2\sqrt{x}} - x^{\frac{1}{2}}}{(x+5)^2} =$$

$$\frac{\frac{5}{2\sqrt{x}} - \frac{1}{2}x^{\frac{1}{2}}}{(x+5)^2} = \frac{\frac{5}{\sqrt{x}} - \sqrt{x}}{2(x+5)^2} = \frac{\frac{5}{\sqrt{x}} - \frac{x}{\sqrt{x}}}{2(x+5)^2} =$$

[Back to Questions](#)

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$$f(x) = \frac{\sqrt{x}}{x+5} = \frac{x^{\frac{1}{2}}}{x+5}.$$

$$f'(x) = \frac{(x+5)(x^{\frac{1}{2}})' - x^{\frac{1}{2}}(x+5)'}{(x+5)^2} = \frac{(x+5)\left(\frac{1}{2}x^{\frac{1}{2}-1}\right) - x^{\frac{1}{2}}(x'+5')}{(x+5)^2} =$$

$$\frac{(x+5)\left(\frac{1}{2}x^{-\frac{1}{2}}\right) - x^{\frac{1}{2}}(1+0)}{(x+5)^2} = \frac{x\frac{1}{2}x^{-\frac{1}{2}} + 5\frac{1}{2}x^{-\frac{1}{2}} - x^{\frac{1}{2}}}{(x+5)^2} = \frac{\frac{1}{2}x^{\frac{1}{2}} + \frac{5}{2\sqrt{x}} - x^{\frac{1}{2}}}{(x+5)^2} =$$

$$\frac{\frac{5}{2\sqrt{x}} - \frac{1}{2}x^{\frac{1}{2}}}{(x+5)^2} = \frac{\frac{5}{\sqrt{x}} - \sqrt{x}}{2(x+5)^2} = \frac{\frac{5}{\sqrt{x}} - \frac{x}{\sqrt{x}}}{2(x+5)^2} = \frac{5-x}{2\sqrt{x}(x+5)^2}$$

[Back to Questions](#)

[Back to Questions](#)

**(7)**  $f(x) = \frac{x}{x+1}.$

$f'(x) =$

[Back to Questions](#)

$$(7) \quad f(x) = \frac{x}{x+1}.$$

$$f'(x) = \frac{(x+1)x' - x(x+1)'}{(x+1)^2} =$$

[Back to Questions](#)

$$(7) \quad f(x) = \frac{x}{x+1}.$$

$$f'(x) = \frac{(x+1)x' - x(x+1)'}{(x+1)^2} = \frac{(x+1)(1) - x(1)}{(x+1)^2} =$$

[Back to Questions](#)

$$(7) \quad f(x) = \frac{x}{x+1}.$$

$$f'(x) = \frac{(x+1)x' - x(x+1)'}{(x+1)^2} = \frac{(x+1)(1) - x(1)}{(x+1)^2} = \frac{x+1-x}{(x+1)^2} =$$

[Back to Questions](#)

$$(7) \quad f(x) = \frac{x}{x+1}.$$

$$f'(x) = \frac{(x+1)x' - x(x+1)'}{(x+1)^2} = \frac{(x+1)(1) - x(1)}{(x+1)^2} = \frac{x+1-x}{(x+1)^2} = \frac{1}{(x+1)^2}$$

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$$(8) \quad f(x) = \frac{x}{\sqrt{x} + 1} = \frac{x}{x^{\frac{1}{2}} + 1}$$

$$f'(x) =$$

[Back to Questions](#)

$$(8) \quad f(x) = \frac{x}{\sqrt{x} + 1} = \frac{x}{x^{\frac{1}{2}} + 1}$$

$$f'(x) = \frac{(x^{\frac{1}{2}} + 1)x' - x(x^{\frac{1}{2}} + 1)'}{(x^{\frac{1}{2}} + 1)^2} =$$

[Back to Questions](#)

$$(8) \quad f(x) = \frac{x}{\sqrt{x} + 1} = \frac{x}{x^{\frac{1}{2}} + 1}$$

$$f'(x) = \frac{(x^{\frac{1}{2}} + 1)x' - x(x^{\frac{1}{2}} + 1)'}{(x^{\frac{1}{2}} + 1)^2} = \frac{(x^{\frac{1}{2}} + 1)(1) - x((x^{\frac{1}{2}})' + 1')}{(x^{\frac{1}{2}} + 1)^2} =$$

[Back to Questions](#)

$$(8) \quad f(x) = \frac{x}{\sqrt{x} + 1} = \frac{x}{x^{\frac{1}{2}} + 1}$$

$$f'(x) = \frac{(x^{\frac{1}{2}} + 1)x' - x(x^{\frac{1}{2}} + 1)'}{(x^{\frac{1}{2}} + 1)^2} = \frac{(x^{\frac{1}{2}} + 1)(1) - x((x^{\frac{1}{2}})' + 1')}{(x^{\frac{1}{2}} + 1)^2} =$$

$$\frac{\sqrt{x} - x\left(\left(\frac{1}{2}x^{\frac{1}{2}-1}\right) + 0\right)}{(x^{\frac{1}{2}} + 1)^2} =$$

[Back to Questions](#)

$$(8) \quad f(x) = \frac{x}{\sqrt{x} + 1} = \frac{x}{x^{\frac{1}{2}} + 1}$$

$$f'(x) = \frac{(x^{\frac{1}{2}} + 1)x' - x(x^{\frac{1}{2}} + 1)'}{(x^{\frac{1}{2}} + 1)^2} = \frac{(x^{\frac{1}{2}} + 1)(1) - x\left(\left(x^{\frac{1}{2}}\right)' + 1'\right)}{(x^{\frac{1}{2}} + 1)^2} =$$

$$\frac{\sqrt{x} - x\left(\left(\frac{1}{2}x^{\frac{1}{2}-1}\right) + 0\right)}{(x^{\frac{1}{2}} + 1)^2} = \frac{\sqrt{x} - x^{\frac{1}{2}}x^{-\frac{1}{2}}}{(x^{\frac{1}{2}} + 1)^2} =$$

[Back to Questions](#)

$$(8) \quad f(x) = \frac{x}{\sqrt{x} + 1} = \frac{x}{x^{\frac{1}{2}} + 1}$$

$$f'(x) = \frac{(x^{\frac{1}{2}} + 1)x' - x(x^{\frac{1}{2}} + 1)'}{(x^{\frac{1}{2}} + 1)^2} = \frac{(x^{\frac{1}{2}} + 1)(1) - x\left(\left(x^{\frac{1}{2}}\right)' + 1'\right)}{(x^{\frac{1}{2}} + 1)^2} =$$

$$\frac{\sqrt{x} - x\left(\left(\frac{1}{2}x^{\frac{1}{2}-1}\right) + 0\right)}{(x^{\frac{1}{2}} + 1)^2} = \frac{\sqrt{x} - x\frac{1}{2}x^{-\frac{1}{2}}}{(x^{\frac{1}{2}} + 1)^2} = \frac{\sqrt{x} - \frac{1}{2}\sqrt{x}}{(x^{\frac{1}{2}} + 1)^2} =$$

[Back to Questions](#)

$$(8) \quad f(x) = \frac{x}{\sqrt{x} + 1} = \frac{x}{x^{\frac{1}{2}} + 1}$$

$$f'(x) = \frac{(x^{\frac{1}{2}} + 1)x' - x(x^{\frac{1}{2}} + 1)'}{(x^{\frac{1}{2}} + 1)^2} = \frac{(x^{\frac{1}{2}} + 1)(1) - x((x^{\frac{1}{2}})' + 1')}{(x^{\frac{1}{2}} + 1)^2} =$$
$$\frac{\sqrt{x} - x\left(\left(\frac{1}{2}x^{\frac{1}{2}-1}\right) + 0\right)}{(x^{\frac{1}{2}} + 1)^2} = \frac{\sqrt{x} - x\frac{1}{2}x^{-\frac{1}{2}}}{(x^{\frac{1}{2}} + 1)^2} = \frac{\sqrt{x} - \frac{1}{2}\sqrt{x}}{(x^{\frac{1}{2}} + 1)^2} = \frac{\frac{1}{2}\sqrt{x}}{(x^{\frac{1}{2}} + 1)^2} =$$

[Back to Questions](#)

$$(8) \quad f(x) = \frac{x}{\sqrt{x} + 1} = \frac{x}{x^{\frac{1}{2}} + 1}$$

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$$\frac{\sqrt{x} - x\left(\left(\frac{1}{2}x^{\frac{1}{2}-1}\right) + 0\right)}{(x^{\frac{1}{2}} + 1)^2} = \frac{\sqrt{x} - x\frac{1}{2}x^{-\frac{1}{2}}}{(x^{\frac{1}{2}} + 1)^2} = \frac{\sqrt{x} - \frac{1}{2}\sqrt{x}}{(x^{\frac{1}{2}} + 1)^2} = \frac{\frac{1}{2}\sqrt{x}}{(x^{\frac{1}{2}} + 1)^2} = \frac{\sqrt{x}}{2(x^{\frac{1}{2}} + 1)^2}$$

[Back to Questions](#)