

Exercises for the Chain Rule

Use the rules of differentiation to find $f(x)'$:

(1) $f(x) = (x + 1)^2$

Solution

(2) $f(x) = (x + 2)^3$

Solution

(3) $f(x) = (x + 3)^4$

Solution

(4) $f(x) = \sqrt{x + 1}$

Solution

(5) $f(x) = \frac{1}{x+1}$

Solution

(6) $f(x) = \frac{1}{\sqrt{x+5}}$

Solution

(7) $f(x) = \left(\frac{x}{x+1}\right)^3$

Solution

(8) $f(x) = x\sqrt{x+1}$

Solution

Solutions

(1)

$$f(x) = (x + 1)^2$$

$$f'(x) =$$

[Back to Questions](#)

Solutions

(1)

$$f(x) = (x + 1)^2$$

$$f'(x) = 2(x + 1)^{2-1}(x + 1)' =$$

[Back to Questions](#)

Solutions

(1)

$$f(x) = (x + 1)^2$$

[Back to Questions](#)

$$f'(x) = 2(x + 1)^{2-1}(x + 1)' = 2(x + 1)$$

(2)

$$f(x) = (x + 2)^3$$

$$f'(x) =$$

[Back to Questions](#)

(2)

$$f(x) = (x + 2)^3$$

$$f'(x) = 3(x + 2)^{3-1}(x + 3)' =$$

[Back to Questions](#)

(2)

$$f(x) = (x + 2)^3$$

$$f'(x) = 3(x + 2)^{3-1}(x + 3)' = 3(x + 2)^2$$

[Back to Questions](#)

(3)

$$f(x) = (x + 3)^4$$

$$f'(x) =$$

[Back to Questions](#)

(3)

$$f(x) = (x + 3)^4$$

$$f'(x) = 4(x + 3)^{4-1}(x + 3)' =$$

[Back to Questions](#)

(3)

$$f(x) = (x + 3)^4$$

$$f'(x) = 4(x + 3)^{4-1}(x + 3)' = 4(x + 3)^3$$

[Back to Questions](#)

(4)

$$f(x) = \sqrt{x+1} = (x+1)^{\frac{1}{2}}. f'(x) =$$

[Back to Questions](#)

(4)

$$f(x) = \sqrt{x+1} = (x+1)^{\frac{1}{2}}. f'(x) = \frac{1}{2}(x+1)^{\frac{1}{2}-1} =$$

[Back to Questions](#)

(4)

$$f(x) = \sqrt{x+1} = (x+1)^{\frac{1}{2}}. f'(x) = \frac{1}{2}(x+1)^{\frac{1}{2}-1} = \frac{1}{2}(x+1)^{-\frac{1}{2}} =$$

[Back to Questions](#)

(4)

$$f(x) = \sqrt{x+1} = (x+1)^{\frac{1}{2}}. f'(x) = \frac{1}{2}(x+1)^{\frac{1}{2}-1} = \frac{1}{2}(x+1)^{-\frac{1}{2}} = \frac{1}{2\sqrt{x+1}}$$

[Back to Questions](#)

(5)

$$f(x) = \frac{1}{x+1} = (x+1)^{-1}$$

$$f'(x) =$$

[Back to Questions](#)

(5)

$$f(x) = \frac{1}{x+1} = (x+1)^{-1}$$

$$f'(x) = (-1)(x+1)^{-1-1} =$$

[Back to Questions](#)

(5)

$$f(x) = \frac{1}{x+1} = (x+1)^{-1}$$

$$f'(x) = (-1)(x+1)^{-1-1} = -\frac{1}{(x+1)^2}$$

[Back to Questions](#)

(6)

$$f(x) = \frac{1}{\sqrt{x+5}} = (x+5)^{-\frac{1}{2}}.$$

$$f'(x) =$$

[Back to Questions](#)

(6)

$$f(x) = \frac{1}{\sqrt{x+5}} = (x+5)^{-\frac{1}{2}}.$$

$$f'(x) = -\frac{1}{2}(x+5)^{-\frac{1}{2}-1} =$$

[Back to Questions](#)

(6)

$$f(x) = \frac{1}{\sqrt{x+5}} = (x+5)^{-\frac{1}{2}}.$$

$$f'(x) = -\frac{1}{2}(x+5)^{-\frac{1}{2}-1} = -\frac{1}{2}(x+5)^{-\frac{3}{2}} =$$

[Back to Questions](#)

(6)

$$f(x) = \frac{1}{\sqrt{x+5}} = (x+5)^{-\frac{1}{2}}.$$

$$f'(x) = -\frac{1}{2}(x+5)^{-\frac{1}{2}-1} = -\frac{1}{2}(x+5)^{-\frac{3}{2}} = -\frac{1}{2(x+5)\sqrt{x+5}}$$

[Back to Questions](#)

[Back to Questions](#)

$$(7) \quad f(x) = \left(\frac{x}{x+1}\right)^3.$$

$$f'(x) =$$

[Back to Questions](#)

$$(7) \quad f(x) = \left(\frac{x}{x+1}\right)^3$$
$$f'(x) = 3 \left(\frac{x}{x+1}\right)^{3-1} \left(\frac{x}{x+1}\right)'$$

[Back to Questions](#)

$$(7) \quad f(x) = \left(\frac{x}{x+1}\right)^3.$$

$$f'(x) = 3 \left(\frac{x}{x+1}\right)^{3-1} \left(\frac{x}{x+1}\right)' = 3 \left(\frac{x}{x+1}\right)^2 \frac{(x+1)x' - x(x+1)'}{(x+1)^2} =$$

[Back to Questions](#)

$$(7) \quad f(x) = \left(\frac{x}{x+1}\right)^3.$$

$$f'(x) = 3 \left(\frac{x}{x+1}\right)^{3-1} \left(\frac{x}{x+1}\right)' = 3 \left(\frac{x}{x+1}\right)^2 \frac{(x+1)x' - x(x+1)'}{(x+1)^2} = 3 \left(\frac{x}{x+1}\right)^2 \frac{(x+1)(1) - x(1)}{(x+1)^2} =$$

[Back to Questions](#)

$$(7) \quad f(x) = \left(\frac{x}{x+1}\right)^3.$$

$$f'(x) = 3 \left(\frac{x}{x+1}\right)^{3-1} \left(\frac{x}{x+1}\right)' = 3 \left(\frac{x}{x+1}\right)^2 \frac{(x+1)x' - x(x+1)'}{(x+1)^2} = 3 \left(\frac{x}{x+1}\right)^2 \frac{(x+1)(1) - x(1)}{(x+1)^2} =$$
$$3 \left(\frac{x}{x+1}\right)^2 \frac{x+1-x}{(x+1)^2} =$$

[Back to Questions](#)

$$(7) \quad f(x) = \left(\frac{x}{x+1}\right)^3.$$

$$\begin{aligned} f'(x) &= 3 \left(\frac{x}{x+1}\right)^{3-1} \left(\frac{x}{x+1}\right)' = 3 \left(\frac{x}{x+1}\right)^2 \frac{(x+1)x' - x(x+1)'}{(x+1)^2} = 3 \left(\frac{x}{x+1}\right)^2 \frac{(x+1)(1) - x(1)}{(x+1)^2} = \\ &= 3 \left(\frac{x}{x+1}\right)^2 \frac{x+1-x}{(x+1)^2} = 3 \left(\frac{x}{x+1}\right)^2 \frac{1}{(x+1)^2} = \end{aligned}$$

[Back to Questions](#)

$$(7) \quad f(x) = \left(\frac{x}{x+1}\right)^3.$$

$$\begin{aligned} f'(x) &= 3 \left(\frac{x}{x+1}\right)^{3-1} \left(\frac{x}{x+1}\right)' = 3 \left(\frac{x}{x+1}\right)^2 \frac{(x+1)x' - x(x+1)'}{(x+1)^2} = 3 \left(\frac{x}{x+1}\right)^2 \frac{(x+1)(1) - x(1)}{(x+1)^2} = \\ &= 3 \left(\frac{x}{x+1}\right)^2 \frac{x+1-x}{(x+1)^2} = 3 \left(\frac{x}{x+1}\right)^2 \frac{1}{(x+1)^2} = \frac{3x^2}{(x+1)^4} \end{aligned}$$

(8) $f(x) = x\sqrt{x+1} = x(x+1)^{\frac{1}{2}}$

$f'(x) =$

[Back to Questions](#)

$$(8) \quad f(x) = x\sqrt{x+1} = x(x+1)^{\frac{1}{2}}$$

$$f'(x) = (x)'(x+1)^{\frac{1}{2}} + x\left((x+1)^{\frac{1}{2}}\right)' =$$

[Back to Questions](#)

$$(8) \quad f(x) = x\sqrt{x+1} = x(x+1)^{\frac{1}{2}}$$

$$f'(x) = (x)'(x+1)^{\frac{1}{2}} + x\left((x+1)^{\frac{1}{2}}\right)' = (1)(x+1)^{\frac{1}{2}} + x\left(\frac{1}{2}(x+1)^{\frac{1}{2}-1}\right) =$$

[Back to Questions](#)

$$(8) \quad f(x) = x\sqrt{x+1} = x(x+1)^{\frac{1}{2}}$$

$$f'(x) = (x)'(x+1)^{\frac{1}{2}} + x\left((x+1)^{\frac{1}{2}}\right)' = (1)(x+1)^{\frac{1}{2}} + x\left(\frac{1}{2}(x+1)^{\frac{1}{2}-1}\right) = \sqrt{x+1} + \frac{x}{2\sqrt{x+1}} =$$

[Back to Questions](#)

$$(8) \quad f(x) = x\sqrt{x+1} = x(x+1)^{\frac{1}{2}}$$

$$f'(x) = (x)'(x+1)^{\frac{1}{2}} + x\left((x+1)^{\frac{1}{2}}\right)' = (1)(x+1)^{\frac{1}{2}} + x\left(\frac{1}{2}(x+1)^{\frac{1}{2}-1}\right) = \sqrt{x+1} + \frac{x}{2\sqrt{x+1}} =$$

$$\sqrt{x+1} \frac{2\sqrt{x+1}}{2\sqrt{x+1}} + \frac{x}{2\sqrt{x+1}} =$$

[Back to Questions](#)

$$(8) \quad f(x) = x\sqrt{x+1} = x(x+1)^{\frac{1}{2}}$$

$$f'(x) = (x)'(x+1)^{\frac{1}{2}} + x\left((x+1)^{\frac{1}{2}}\right)' = (1)(x+1)^{\frac{1}{2}} + x\left(\frac{1}{2}(x+1)^{\frac{1}{2}-1}\right) = \sqrt{x+1} + \frac{x}{2\sqrt{x+1}} =$$

$$\sqrt{x+1} \frac{2\sqrt{x+1}}{2\sqrt{x+1}} + \frac{x}{2\sqrt{x+1}} = \frac{2(x+1)}{2\sqrt{x+1}} + \frac{x}{2\sqrt{x+1}} =$$

[Back to Questions](#)

$$(8) \quad f(x) = x\sqrt{x+1} = x(x+1)^{\frac{1}{2}}$$

$$f'(x) = (x)'(x+1)^{\frac{1}{2}} + x\left((x+1)^{\frac{1}{2}}\right)' = (1)(x+1)^{\frac{1}{2}} + x\left(\frac{1}{2}(x+1)^{\frac{1}{2}-1}\right) = \sqrt{x+1} + \frac{x}{2\sqrt{x+1}} =$$

$$\sqrt{x+1} \frac{2\sqrt{x+1}}{2\sqrt{x+1}} + \frac{x}{2\sqrt{x+1}} = \frac{2(x+1)}{2\sqrt{x+1}} + \frac{x}{2\sqrt{x+1}} = \frac{3x+2}{2\sqrt{x+1}}$$

[Back to Questions](#)
