Completing the Square

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The basic idea is that we rewrite $ax^2 + bx + c$ in the form

$$a[(x + h)^2 + k].$$

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$(x + 1)^2 + 1$

But how did we know to add 0 in the form $1 - 1$? The answer is that 1 is one-half of the coefficient 2 of x in the expression $x^2 + 2x + 2$.

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$$\left(x + \frac{9}{2}\right)^2 - \frac{61}{4}$$

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Completing the Square-8

$$a\left[\overbrace{x^2+2\left(\frac{b}{2a}\right)x+\left(\frac{b}{2a}\right)^2}^{\left(x+\frac{b}{2a}\right)^2}-\left(\frac{b}{2a}\right)^2+\frac{c}{a}\right]=$$

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$$3\left[\overbrace{x^2+2\left(\frac{5}{6}\right)x+\left(\frac{5}{6}\right)^2}^{\left(x+\frac{5}{6}\right)^2}-\left(\frac{5}{6}\right)^2+\frac{2}{3}\left(\frac{12}{12}\right)\right]=$$

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$$3\left[\left(x + \frac{5}{6}\right)^2 - \frac{25}{36} + \frac{24}{36}\right] =$$

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$$3\left[\left(x+\frac{5}{6}\right)^2-\frac{25}{36}+\frac{24}{36}\right]=3\left[\left(x+\frac{5}{6}\right)^2-\frac{1}{36}\right]=$$

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$$(x+1)(3x+2)$$

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$$ax^2 + bx + c = a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2}\right]$$

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x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} = \pm\frac{\sqrt{b^2 - 4ac}}{2a} \quad &\iff
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$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} = \pm\frac{\sqrt{b^2 - 4ac}}{2a} \iff$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} =$$

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&x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} = \pm\frac{\sqrt{b^2 - 4ac}}{2a} && \iff \\
&x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
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&x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} = \pm\frac{\sqrt{b^2 - 4ac}}{2a} && \iff \\
&x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\end{aligned}
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which is the familiar Quadratic Formula for the roots of the general quadratic equation.