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$\frac{(9+x^2)(3-x)(3+x)}{(x^2+3)(3+x)}=\frac{(9+x^2)(3-x)}{(x^2+3)}=$

Algebra of Fractions-4

$$\frac{9(3-x)+x^2(3-x)}{x^2+3} =$$

$$\frac{9(3-x)+x^2(3-x)}{x^2+3} =$$

$$\frac{9(3)-9(x)+x^2(3)-x^2(x)}{x^2+3} =$$

$$\frac{9(3-x)+x^2(3-x)}{x^2+3} =$$

$$\frac{9(3)-9(x)+x^2(3)-x^2(x)}{x^2+3} = \frac{27-9x+3x^2-x^3}{x^2+3}$$

Algebra of Fractions-5

Suppose $c = x$ and $d = 1$, so that $\frac{c}{d} = \frac{x}{1} = x$. Then

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so multiplying a fraction by x has the same effect as multiplying the numerator of the fraction by x.

Example 4: $\dfrac{2x+3}{3x+1}\pi =$

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Addition

Case 1: Same Denominator:

To add two fractions with the same denominator, *add the numerators together*, i.e.,

$$\frac{a}{h} + \frac{b}{h} = \frac{a+b}{h}$$

(Consider: two-sixths of a pizza plus one-sixth of a pizza equals three-sixths, or one-half of a pizza.)

Case 2: Different Denominators:

For fractions with different denominators, say $\frac{a}{b}$ and $\frac{c}{d}$, we must first alter the fractions to have a **common denominator**, say $h = bd$. We do this by multiplications by 1, where we write 1 as an appropriate fraction:

$$\frac{a}{b} + \frac{c}{d} =$$

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$$\frac{a}{b} + \frac{c}{d} = \frac{a}{b} \cdot 1 + 1 \cdot \frac{c}{d} =$$

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$$\frac{a}{b} + \frac{c}{d} = \frac{a}{b} \cdot 1 + 1 \cdot \frac{c}{d} = \left(\frac{a}{b}\right) \cdot \left(\frac{d}{d}\right) + \left(\frac{b}{b}\right) \cdot \left(\frac{c}{d}\right) =$$

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We have thus derived the basic addition formula:

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

The formula is easily remembered as “cross-multiplication”:

$$\frac{a}{b} \bowtie \frac{c}{d} \rightarrow \frac{ad+bc}{bd}$$

Example 5:

$$\frac{3x+2}{5} + \frac{7x+1}{8} =$$

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$$\frac{a}{b} \bowtie \frac{c}{d} \to \frac{ad + bc}{bd}$$

Example 5:

$$\frac{3x + 2}{5} + \frac{7x + 1}{8} =$$

$$\frac{(3x + 2)8 + 5(7x + 1)}{5(8)} =$$

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$$\frac{(3x+2)8 + 5(7x+1)}{5(8)} =$$

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$$\frac{10x^3}{15x} + \frac{18}{15x} =$$

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$$\frac{10x^3}{15x} + \frac{18}{15x} = \frac{10x^3 + 18}{15x}$$

Factor if You Can

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Calculations with fractions can be simplified by factoring if there are common factors:

Example 7:

$$\frac{x^2 - 1}{15x + 3} + \frac{x^2 - x - 2}{5x + 1} =$$

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Example 7:

$$\frac{x^2-1}{15x+3}+\frac{x^2-x-2}{5x+1}=\frac{(x+1)(x-1)}{3(5x+1)}+\frac{(x+1)(x-2)}{5x+1}=$$

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$$\frac{x+1}{5x+1}\left(\frac{x-1}{3}+\frac{3(x-2)}{3}\right)=$$

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$$\frac{x+1}{5x+1}\left(\frac{4x-7}{3}\right)=$$

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$$\frac{x+1}{5x+1}\left(\frac{4x-7}{3}\right)=\frac{(x+1)(4x-7)}{3(5x+1)}$$

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Solution

$$\frac{h+5}{5-h}+\frac{h}{1+h}=\frac{(h+5)(1+h)+(5-h)h}{(5-h)(1+h)}=$$

$$\frac{h+h^2+5h+5+5h-h^2}{5+5h-h-h^2}=$$

Example 8: Combine $\dfrac{h+5}{5-h} + \dfrac{h}{1+h}$ into one fraction.

Solution

$$\frac{h+5}{5-h} + \frac{h}{1+h} = \frac{(h+5)(1+h) + (5-h)h}{(5-h)(1+h)} =$$

$$\frac{h + h^2 + 5h + 5 + 5h - h^2}{5 + 5h - h - h^2} = \frac{11h+5}{5+4h-h^2}$$

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$$\frac{x+5h+3x-3h}{x-h}=$$

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$$\frac{x+5h}{x-h}+3=\frac{x+5h}{x-h}+\frac{3}{1}=\frac{(x+5h)+3(x-h)}{x-h}=$$

$$\frac{x+5h+3x-3h}{x-h}=\frac{4x+2h}{x-h}$$

Note: In the basic equation

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

we used the denominator $h = bd$ because it was a **common denominator** for both $\frac{a}{b}$ and $\frac{c}{d}$; however, in many situations, a smaller common denominator will exist, and computations will be greatly simplified if we use it instead. Here is a good example:

Example 10: Combine

$$\frac{3-2x^2}{3a^4(x+2)^3(x-1)^3}+\frac{x+1}{6a^3(x+2)^3(x-1)^2}$$

Solution:

We could use the common denominator

$h=[3a^4(x+2)^3(x-1)^3]\,[6a^3(x+2)^3(x-1)^2]=$
$12a^7(x+2)^6(x-1)^5$

...but that would be inefficient: we notice that there are many common factors. The easier way is to factor out the common factors and then add the simplified fractions:

$$\frac{3-2x^2}{3a^4(x+2)^3(x-1)^3}+\frac{x+1}{6a^3(x+2)^3(x-1)^2}=$$

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$$\frac{3-2x^2}{3a^4(x+2)^3(x-1)^3}+\frac{x+1}{6a^3(x+2)^3(x-1)^2}=$$

$$\frac{1}{3a^3(x+2)^3(x-1)^2}\left[\frac{3-2x^2}{a(x-1)}+\frac{x+1}{2}\right]=$$

Example 10: Combine

$$\frac{3-2x^2}{3a^4(x+2)^3(x-1)^3}+\frac{x+1}{6a^3(x+2)^3(x-1)^2}$$

Solution:

We could use the common denominator

$h = [3a^4(x+2)^3(x-1)^3]\,[6a^3(x+2)^3(x-1)^2] =$
$12a^7(x+2)^6(x-1)^5$

...but that would be inefficient: we notice that there are many common factors. The easier way is to factor out the common factors and then add the simplified fractions:

$$\frac{3-2x^2}{3a^4(x+2)^3(x-1)^3}+\frac{x+1}{6a^3(x+2)^3(x-1)^2}=$$

$$\frac{1}{3a^3(x+2)^3(x-1)^2}\left[\frac{3-2x^2}{a(x-1)}+\frac{x+1}{2}\right]=$$

$$\frac{1}{3a^3(x+2)^3(x-1)^2}\left[\frac{(3-2x^2)2+a(x-1)(x+1)}{a(x-1)(2)}\right]=$$

$$\frac{1}{3a^3(x+2)^3(x-1)^2}\left[\frac{(3-2x^2)2+a(x-1)(x+1)}{a(x-1)(2)}\right]=$$

$$\frac{1}{3a^3(x+2)^3(x-1)^2}\left[\frac{6-4x^2+ax^2-a}{2a(x-1)}\right]=$$

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$$\frac{1}{3a^3(x+2)^3(x-1)^2}\left[\frac{6-4x^2+ax^2-a}{2a(x-1)}\right]=$$

$$\frac{(a-4)x^2+6-a}{6a^4(x+2)^3(x-1)^3}$$

It is good practice to perform side calculations: after the first line, it is better to add the simplified fractions in a separate calculation: we would write

$$\frac{3-2x^2}{a(x-1)}+\frac{x+1}{2}=$$

$$\frac{3-2x^2}{a(x-1)}+\frac{x+1}{2}=\frac{(3-2x^2)2+a(x-1)(x+1)}{a(x-1)(2)}=$$

$$\frac{3-2x^2}{a(x-1)} + \frac{x+1}{2} = \frac{(3-2x^2)2 + a(x-1)(x+1)}{a(x-1)(2)} =$$

$$\frac{6-4x^2+ax^2-a}{2a(x-1)} =$$

and then we would write:

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and then we would write:

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$$\frac{3-2x^2}{a(x-1)} + \frac{x+1}{2} = \frac{(3-2x^2)2 + a(x-1)(x+1)}{a(x-1)(2)} =$$

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and then we would write:

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$$\frac{1}{3a^3(x+2)^3(x-1)^2}\left[\frac{6-4x^2+ax^2-a}{2a(x-1)}\right] =$$

$$\frac{3-2x^2}{a(x-1)}+\frac{x+1}{2}=\frac{(3-2x^2)2+a(x-1)(x+1)}{a(x-1)(2)}=$$
$$\frac{6-4x^2+ax^2-a}{2a(x-1)}=$$

and then we would write:

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$$\frac{1}{3a^3(x+2)^3(x-1)^2}\left[\frac{6-4x^2+ax^2-a}{2a(x-1)}\right]=$$

$$\frac{(a-4)x^2+6-a}{6a^4(x+2)^3(x-1)^3}$$

Subtraction

If you know how to add fractions then you know how to subtract them; the trick is to convert $-\left(\frac{c}{d}\right)$ into $\frac{-c}{d}$:

$$\frac{a}{b} - \frac{c}{d} =$$

Subtraction

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Division

To divide $\frac{a}{b}$ by $\frac{c}{d}$, we remember that $\frac{c}{d} \cdot \frac{d}{c} = \frac{cd}{cd} = 1$,

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Thus, to simplify the quotient of two fractions, we multiply

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Example 13: Simplify $\dfrac{\dfrac{a+b}{a-b}+\dfrac{a-b}{a+b}}{\dfrac{a^2+b^2}{2}}$

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$$\frac{\frac{2a^2+2b^2}{(a^2-b^2)}}{\frac{a^2+b^2}{2}}=\left(\frac{2(a^2+b^2)}{(a^2-b^2)}\right)\left(\frac{2}{a^2+b^2}\right)=$$

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