Some Basic Algebra Facts

Commutativity: $a + b = b + a$ and $ab = ba$

Associativity: $a + (b + c) = (a + b) + c$ and $a(bc) = (ab)c$

Adding or Subtracting 0: $a + 0 = 0 + a = a$ and
$a - 0 = -0 + a = a$

Multiplying by 1: $a \cdot 1 = 1 \cdot a = a$

Simple Factoring:

$a^2 + ab = a \cdot a + a \cdot b = a(a + b) = (a + b)a$

and $a^2 - ab = a \cdot a - a \cdot b = a(a - b) = (a - b)a$

In particular, if $b = 1$, we have:

$a^2 + a = a \cdot a + a \cdot 1 = a(a + 1)$

and $a^2 - a = a \cdot a - a \cdot 1 = a(a - 1)$

$a^2 - b^2 = (a + b)(a - b)$

If $b = 1$, we have $a^2 - 1^2 = (a + 1)(a - 1)$, but since $1^2 = 1$, we can write $a^2 - 1 = (a + 1)(a - 1)$.

If $a = 1$, we get $1^2 - b^2 = (1 - b)(1 + b)$, so can write $1 - b^2 = (1 - b)(1 + b)$.

Example 1 Factor $x^3 - x$

Solution: $x^3 - x = x \cdot x^2 - x \cdot 1 = x(x^2 - 1) =$ $x(x + 1)(x - 1)$

Example 2 Factor $x^3 + x$

Solution: $x^3 + x = x \cdot x^2 + x \cdot 1 = x(x^2 + 1)$

Example 3 Factor $x^4 - x^2$

Solution: $x^4 - x^2 = x^2 \cdot x^2 - x^2 \cdot 1 = x^2(x^2 - 1) = x^2(x + 1)(x - 1)$
