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Examples of *non*-polynomials are:

$$x^2 + \sqrt{x} \quad (\text{because } \sqrt{x} = x^{\frac{1}{2}} \text{ is a } \textit{non-integer} \text{ power of } x)$$

$$3 + \frac{1}{x} \quad (\text{because } \frac{1}{x} = x^{-1} \text{ is a } \textit{negative} \text{ power of } x)$$

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$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

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$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

where $a_n, a_{n-1}, \dots, a_1, a_0$ are constants, called the **coefficients** of $p(x)$.

The **degree** of a polynomial is the highest power of x that it contains. For example, $x^4 - 1$ is of degree 4, while $\sqrt{3}$ is of degree 0. Polynomials of low degrees have special names:

Polynomials-2

degree	form	name
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When manipulating polynomials it is always best to write them with the highest degree term on the left and the lowest degree term on the right, and with the terms arranged in descending order of degree from left to right.

Multiplication of Algebraic Expressions

Most calculus students are comfortable with the multiplication of simple expressions such as

$$(a + b)(c + d) = ac + ad + bc + bd$$

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For multiplication of longer expressions a person can always resort, if confused, to “long multiplication,” as we now illustrate.

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Solution: An inefficient way of carrying out this calculation would be:

$$(3x + 4y - 1) \times (x - 2y + 2) =$$

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The procedure is just that of ordinary multiplication of numbers; the only difference is in the placement of “like terms” underneath each other to aid in the final addition.

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Multiplication of polynomials can be simplified by “detaching the coefficients” from the powers of x : we repeat the above calculation:

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Powers of $a + b$ — Pascal's Triangle

This is an easy way of finding integer powers of $a + b$. The triangle is bordered by 1's and has the property that every interior entry is the sum of the two entries just above it:

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$$n = 1: (a + b)^1 = 1 \cdot a^1 b^0 + 1 \cdot a^0 b^1 = a + b$$

Powers of $a + b$ — Pascal's Triangle

This is an easy way of finding integer powers of $a + b$. The triangle is bordered by 1's and has the property that every interior entry is the sum of the two entries just above it:

$n = 0$					1										
$n = 1$				1		1									
$n = 2$			1		2		1								
$n = 3$			1		3		3		1						
$n = 4$		1		4		6		4		1					
$n = 5$		1		5		10		10		5		1			
$n = 6$	1		6		15		20		15		6		1		
$n = 7$	1		7		21		35		35		21		7		1

The n^{th} row of the triangle contains the coefficients of $(a + b)^n$:

$$n = 0: (a + b)^0 = 1 \cdot a^0 b^0 = 1$$

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$$n = 2: (a + b)^2 = 1 \cdot a^2 b^0 + 2 \cdot a^1 b^1 + 1 \cdot a^0 b^2 = a^2 + 2ab + b^2$$

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$n = 4$			1		4		6		4		1				
$n = 5$			1		5		10		10		5		1		
$n = 6$		1		6		15		20		15		6		1	
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$$n = 2: (a + b)^2 = 1 \cdot a^2 b^0 + 2 \cdot a^1 b^1 + 1 \cdot a^0 b^2 = a^2 + 2ab + b^2$$

$$n = 3: (a + b)^3 = 1 \cdot a^3 b^0 + 3 \cdot a^2 b^1 + 3 \cdot a^1 b^2 + 1 \cdot a^0 b^3 = a^3 + 3a^2 b + 3ab^2 + b^3$$

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$n = 5$		1		5		10		10		5	1			
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$$n = 3: (a + b)^3 = 1 \cdot a^3 b^0 + 3 \cdot a^2 b^1 + 3 \cdot a^1 b^2 + 1 \cdot a^0 b^3 = a^3 + 3a^2 b + 3ab^2 + b^3$$

$$n = 4: (a + b)^4 = 1 \cdot a^4 b^0 + 4 \cdot a^3 b^1 + 6 \cdot a^2 b^2 + 4 \cdot a^1 b^3 + 1 \cdot a^0 b^4 = a^4 + 4a^3 b + 6a^2 b^2 + 4a^1 b^3 + b^4$$

$$n = 5: (a + b)^5 = 1 \cdot a^5b^0 + 5 \cdot a^4b^1 + 10 \cdot a^3b^2 + 10 \cdot a^2b^3 + 5 \cdot a^1b^4 + 1 \cdot a^0b^5 = \\ a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

and so on.

We can build another useful triangle:

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We can build another useful triangle:

$$(a + b)^0 =$$

1

Division

The division of one algebraic expression by another frequently gives people difficulty. However, the technique of “long division” is quite important and is similar to the usual “long division” of decimal numbers.

Example 3:

Divide $x^3 + x^2 + x - 3$ by $x - 1$, i.e., simplify $\frac{x^3 + x^2 + x - 3}{x - 1}$

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divisor | dividend

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$$\text{divisor} \overline{) \text{dividend}} \qquad x - 1 \overline{) x^3 + x^2 + x - 3}$$

Notice that both terms are written with the powers of x in *descending order*.

1. Take the first term x in the divisor and divide it into the first term x^3 in the dividend;

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$$\begin{array}{r}
 x^2 \\
 \hline
 x - 1 \quad \left| \quad x^3 \quad +x^2 \quad +x \quad -3
 \end{array}
 \quad \left(= \frac{x^3}{x} \right)$$

Now multiply the divisor $(x - 1)$ by x^2 , place the result $(x^3 - x^2)$ under the dividend (with correct positioning of the powers of x), and subtract:

$$\begin{array}{r}
 x^2 \\
 \hline
 x - 1 \quad \left| \quad x^3 \quad +x^2 \quad +x \quad -3 \\
 \underline{x^3 \quad -x^2} \\

 \end{array}
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 \underline{x^3 - x^2} \\
 2x^2 + x - 3 \\
 \underline{2x^2 - 2x + 3} \\
 3x - 6
 \end{array}
 \quad \begin{array}{l}
 (= x^2(x-1)) \\
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Note that the bottom term is the dividend minus x^2 times the divisor:

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 x^2 \\
 \hline
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 \hline
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 \end{array}
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Note that the bottom term is the dividend minus x^2 times the divisor:

$$2x^2 + x - 3 = (x^3 + x^2 + x - 3) - x^2(x - 1)$$

It will be referred to as the “new” or “second” dividend”.

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 x^2 \\
 \hline
 x - 1 \quad | \quad x^3 + x^2 + x - 3 \\
 \quad \quad \quad x^3 - x^2 \\
 \hline
 \quad \quad \quad 2x^2 + x - 3
 \end{array}
 \quad (= x^2(x - 1))$$

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$$x^2$$

$$x^2 \quad \square$$

$$x^2 + 2x$$

$$x^2 + 2x$$



$$\begin{array}{r}
 x^2 + 2x \\
 \left(= \frac{2x^2}{x} \right) \\
 x - 1 \overline{) \begin{array}{r} x^3 + x^2 + x - 3 \\ x^3 - x^2 \\ \hline 2x^2 + x - 3 \end{array}
 \end{array}$$

$$\begin{array}{r}
 x^2 + 2x \\
 \left(= \frac{2x^2}{x} \right) \\
 \hline
 x - 1 \quad \left| \begin{array}{r}
 x^3 + x^2 + x - 3 \\
 x^3 - x^2 \\
 \hline
 2x^2 + x - 3
 \end{array} \right.
 \end{array}$$

Next we multiply the divisor $(x - 1)$ by $2x$ and place the resulting $2x^2 - 2x$ under the new dividend:

$$\begin{array}{r}
 x^2 + 2x \\
 \hline
 x - 1 \quad \left| \begin{array}{r}
 x^3 + x^2 + x - 3 \\
 x^3 - x^2 \\
 \hline
 2x^2 + x - 3
 \end{array} \right.
 \end{array}$$

$$\begin{array}{r}
 x^2 + 2x \\
 \hline
 x-1 \overline{) x^3 + x^2 + x - 3} \\
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 \underline{x^3 - x^2} \\
 2x^2 + x - 3 \\
 \underline{2x^2 - 2x} \\
 3x - 3
 \end{array}$$

$$\begin{array}{r}
 x^2 + 2x \\
 \hline
 x-1 \left| \begin{array}{r} x^3 + x^2 + x - 3 \\ x^3 - x^2 \\ \hline 2x^2 + x - 3 \end{array} \right.
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 x^2 + 2x \\
 \hline
 x-1 \left| \begin{array}{r} x^3 + x^2 + x - 3 \\ x^3 - x^2 \\ \hline 2x^2 + x - 3 \\ 2x^2 - 2x \end{array} \right.
 \end{array}$$

$$\begin{array}{r}
 x^2 + 2x \\
 \hline
 x - 1 \overline{) x^3 + x^2 + x - 3} \\
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$$\begin{array}{r}
 x^2 + 2x \\
 \hline
 x - 1 \overline{) x^3 + x^2 + x - 3} \\
 \underline{x^3 - x^2} \\
 2x^2 + x - 3 \\
 \underline{2x^2 - 2x} \\
 3x - 3
 \end{array}
 \quad = 2x(x - 1)$$

Next we subtract:

$$\begin{array}{r}
 x^2 + 2x \\
 \hline
 x - 1 \overline{) x^3 + x^2 + x - 3} \\
 \underline{x^3 - x^2} \\
 2x^2 + x - 3 \\
 \underline{2x^2 - 2x} \\
 3x - 3
 \end{array}$$

$$\begin{array}{r}
 x^2 + 2x \\
 \hline
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 x^2 + 2x \\
 \hline
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 x^2 + 2x \\
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 x^2 + 2x \\
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 2x^2 + x - 3 \\
 \underline{2x^2 - 2x} \\
 3x - 3
 \end{array}$$

$$\begin{array}{r}
 x^2 + 2x \\
 \hline
 x - 1 \left| \begin{array}{r} x^3 + x^2 + x - 3 \\ x^3 - x^2 \\ \hline 2x^2 + x - 3 \end{array} \right.
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 x^2 + 2x \\
 \hline
 x - 1 \left| \begin{array}{r} x^3 + x^2 + x - 3 \\ x^3 - x^2 \\ \hline 2x^2 + x - 3 \\ 2x^2 - 2x \\ \hline 3x - 3 \end{array} \right.
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 x^2 + 2x \\
 \hline
 x - 1 \left| \begin{array}{r} x^3 + x^2 + x - 3 \\ x^3 - x^2 \\ \hline 2x^2 + x - 3 \\ 2x^2 - 2x \\ \hline 3x - 3 \end{array} \right.
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 \hline
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 x^2 + 2x \\
 \hline
 x - 1 \left| \begin{array}{r} x^3 + x^2 + x - 3 \\ x^3 - x^2 \\ \hline 2x^2 + x - 3 \\ 2x^2 - 2x \\ \hline 3x - 3 \end{array} \right.
 \end{array}
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Next we subtract:

$$\begin{array}{r}
 x^2 + 2x \\
 \hline
 x - 1 \left| \begin{array}{r} x^3 + x^2 + x - 3 \\ x^3 - x^2 \\ \hline 2x^2 + x - 3 \\ 2x^2 - 2x \\ \hline 3x - 3 \end{array} \right.
 \end{array}
 \quad = 2x^2 + x - 3 - (2x^2 - 2x)$$

Note that the bottom term is now the dividend minus $x^2 + 2x$ times the divisor:

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 \end{array}
 \quad \left(= \frac{2x^2}{x} \right)$$

Next we multiply the divisor $(x - 1)$ by $2x$ and place the resulting $2x^2 - 2x$ under the new dividend:

$$\begin{array}{r}
 x^2 + 2x \\
 \hline
 x - 1 \left| \begin{array}{r} x^3 + x^2 + x - 3 \\ x^3 - x^2 \\ \hline 2x^2 + x - 3 \\ 2x^2 - 2x \\ \hline + 3x - 3 \end{array} \right.
 \end{array}
 \quad = 2x(x - 1)$$

Next we subtract:

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$$3x - 3 = (x^3 + x^2 + x - 3) - (x^2 + 2x)(x - 1)$$

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Note that the bottom term is now the dividend minus $x^2 + 2x$ times the divisor:

$$3x - 3 = (x^3 + x^2 + x - 3) - (x^2 + 2x)(x - 1)$$

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$$x^2 + 2x$$


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$$x^2 + 2x \quad \blacksquare$$

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Detached Coefficients

As with multiplication, division of polynomials can be simplified by “detaching the coefficients” from the powers of x : we repeat the above calculation:

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$$\begin{array}{r|rrrr}
 x^1 & x^0 & & & & & & \\
 \hline
 & & x^3 & x^2 & x^1 & x^0 & & \\
 & & & 1 & 2 & 3 & & \\
 \hline
 1 & -1 & | & 1 & 1 & 1 & -3 & \\
 & & & 1 & -1 & & & \\
 \hline
 & & & & 2 & 1 & -3 & \\
 & & & & 2 & -2 & & \\
 \hline
 & & & & & 3 & -3 & \\
 & & & & & 3 & -3 & \\
 \hline
 & & & & & & & 0
 \end{array}$$

which means exactly the same thing as the previous calculation!

Example 4

Divide $x^3 + 2x^2 + 3x + 2$ by $x^2 + 1$, i.e., simplify $\frac{x^3 + 2x^2 + 3x + 2}{x^2 + 1}$.

Solution: Again we set up the divisor and dividend with the terms in decreasing powers of x .

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1. Divide x^2 into x^3 to get x ;

Solution: Again we set up the divisor and dividend with the terms in decreasing powers of x .

$$x^2 + 1 \quad \overline{) \quad x^3 + 2x^2 + 3x + 2}$$

1. Divide x^2 into x^3 to get x ; multiply $x^2 + 1$ by x

Solution: Again we set up the divisor and dividend with the terms in decreasing powers of x .

$$x^2 + 1 \overline{) x^3 + 2x^2 + 3x + 2}$$

1. Divide x^2 into x^3 to get x ; multiply $x^2 + 1$ by x and subtract the result from the dividend:

$$\begin{array}{r} x \\ x^2 + 1 \overline{) x^3 + 2x^2 + 3x + 2} \\ \underline{x^3 + x} \\ 2x^2 + 2x + 2 \end{array}$$

Solution: Again we set up the divisor and dividend with the terms in decreasing powers of x .

$$x^2 + 1 \overline{) x^3 + 2x^2 + 3x + 2}$$

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Note that the bottom term is the dividend minus x times the divisor:

$$2x^2 + 2x + 2 = (x^3 + 2x^2 + 3x + 2) - x(x^2 + 1)$$

Solution: Again we set up the divisor and dividend with the terms in decreasing powers of x .

$$x^2 + 1 \overline{) x^3 + 2x^2 + 3x + 2}$$

1. Divide x^2 into x^3 to get x ; multiply $x^2 + 1$ by x and subtract the result from the dividend:

$$\begin{array}{r} x \\ x^2 + 1 \overline{) x^3 + 2x^2 + 3x + 2} \\ \underline{x^3 + x} \\ 2x^2 + 2x + 2 \end{array}$$

Note that the bottom term is the dividend minus x times the divisor:

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2. Divide x^2 into $2x^2$ to get 2;

Solution: Again we set up the divisor and dividend with the terms in decreasing powers of x .

$$x^2 + 1 \overline{) x^3 + 2x^2 + 3x + 2}$$

1. Divide x^2 into x^3 to get x ; multiply $x^2 + 1$ by x and subtract the result from the dividend:

$$\begin{array}{r} x \\ x^2 + 1 \overline{) x^3 + 2x^2 + 3x + 2} \\ \underline{x^3 + x} \\ 2x^2 + 2x + 2 \end{array}$$

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Solution: Again we set up the divisor and dividend with the terms in decreasing powers of x .

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$$\begin{array}{r} x \\ x^2 + 1 \overline{) x^3 + 2x^2 + 3x + 2} \\ \underline{x^3 + x} \\ 2x^2 + 2x + 2 \end{array}$$

Note that the bottom term is the dividend minus x times the divisor:

$$2x^2 + 2x + 2 = (x^3 + 2x^2 + 3x + 2) - x(x^2 + 1)$$

2. Divide x^2 into $2x^2$ to get 2; multiply $x^2 + 1$ by 2 and subtract the result from the “new dividend”.

$$\begin{array}{r} x + 2 \\ x^2 + 1 \overline{) x^3 + 2x^2 + 3x + 2} \\ \underline{x^3 + x} \\ 2x^2 + 2x + 2 \\ \underline{2x^2 + 2} \\ 2x \end{array}$$

Solution: Again we set up the divisor and dividend with the terms in decreasing powers of x .

$$x^2 + 1 \overline{) x^3 + 2x^2 + 3x + 2}$$

1. Divide x^2 into x^3 to get x ; multiply $x^2 + 1$ by x and subtract the result from the dividend:

$$\begin{array}{r} x \\ x^2 + 1 \overline{) x^3 + 2x^2 + 3x + 2} \\ \underline{x^3 + x} \\ 2x^2 + 2x + 2 \end{array}$$

Note that the bottom term is the dividend minus x times the divisor:

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Now the bottom term is the dividend minus $x + 2$ times the divisor:

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$$x^2 + 1 \overline{) x^3 + 2x^2 + 3x + 2}$$

1. Divide x^2 into x^3 to get x ; multiply $x^2 + 1$ by x and subtract the result from the dividend:

$$\begin{array}{r} x \\ x^2 + 1 \overline{) x^3 + 2x^2 + 3x + 2} \\ \underline{x^3 + x} \\ 2x^2 + 2x + 2 \end{array}$$

Note that the bottom term is the dividend minus x times the divisor:

$$2x^2 + 2x + 2 = (x^3 + 2x^2 + 3x + 2) - x(x^2 + 1)$$

2. Divide x^2 into $2x^2$ to get 2; multiply $x^2 + 1$ by 2 and subtract the result from the “new dividend”.

$$\begin{array}{r} x + 2 \\ x^2 + 1 \overline{) x^3 + 2x^2 + 3x + 2} \\ \underline{x^3 + x} \\ 2x^2 + 2x + 2 \\ \underline{2x^2 + 2x + 2} \\ 2x \end{array}$$

Now the bottom term is the dividend minus $x + 2$ times the divisor: $2x = (x^3 + 2x^2 + 3x + 2) - (x + 2)(x^2 + 1)$

There is a major difference from the previous example:

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$$\frac{x^3 + 2x^2 + 3x + 2}{x^2 + 1} = x + 2 + \frac{2x}{x^2 + 1}$$

The detached coefficient calculation is:

x^2	x^1	x^0		x^3	x^2	x^1	x^0
					1	2	
1	0	1		1	2	3	2
				1	0	1	
					2	2	2
					2	0	2
					2	0	



Example 5 Divide $2t^2 - t + t^3$ by $t + 1$

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Solution: We have to rewrite the dividend as $t^3 + 2t^2 - t$ so that the powers of t are in decreasing order.

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$$\begin{array}{r}
 \begin{array}{cccc|cccc}
 t^1 & t^0 & | & t^3 & t^2 & t^1 & t^0 & \\
 \hline
 & & & & 1 & 1 & -2 & \\
 \hline
 1 & 1 & | & 1 & 2 & -1 & & \\
 & & & 1 & 1 & & & \\
 \hline
 & & & & 1 & -1 & & \\
 & & & & 1 & 1 & & \\
 \hline
 & & & & & -2 & & \\
 & & & & & -2 & -2 & \\
 \hline
 & & & & & & &
 \end{array}
 \end{array}$$

The detached coefficient calculation is:

$$\begin{array}{r}
 \begin{array}{cccc|cccc}
 t^1 & t^0 & & & t^3 & t^2 & t^1 & t^0 \\
 \hline
 & & & & & 1 & 1 & -2 \\
 \hline
 1 & 1 & & & 1 & 2 & -1 & \\
 & & & & 1 & 1 & & \\
 \hline
 & & & & & 1 & -1 & \\
 & & & & & 1 & 1 & \\
 \hline
 & & & & & & -2 & \\
 & & & & & & -2 & -2 \\
 \hline
 & & & & & & & 2
 \end{array}
 \end{array}$$