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$$x^4 - 16 = (x - 2)(x + 2)(x^2 + 4)$$

Factoring a polynomial is not always a pleasant or easy operation, but it is important: because in addition to telling us where a polynomial is zero, it is the tool we will need to determine exactly where a polynomial is positive or negative. This in turn will vital to the sketching of the graphs of polynomials, one of the highlights of this course.

We will study the procedure in some depth, first for quadratic (or degree 2) polynomials, and then for general polynomials.

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Thus $2x^2 + x - 6 = 2(x - (-2))\left(x - \frac{3}{2}\right)$

which appears to be different from the factorization $2x^2 + x - 6 = (2x + (-3))(x + 2) = (2x - 3)(x + 2)$ that we found by “inspection”!

However, we note that

$$2(x - (-2))\left(x - \frac{3}{2}\right) = (x + 2)2\left(x - \frac{3}{2}\right) =$$

$$(x + 2)\left(2x - 2\frac{3}{2}\right) = (x + 2)(2x - 3) = (2x - 3)(x + 2),$$

so we have two different but equivalent factorizations of the same polynomial.

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You can (and perhaps *should*) always check a factorization by multiplying the terms to see if you get what you started with.

$$3x^2 - 5x + 1 = 3\left(x - \frac{5 + \sqrt{13}}{6}\right)\left(x - \frac{5 - \sqrt{13}}{6}\right) =$$

$$3\left[x^2 - \left(\frac{5 + \sqrt{13}}{6} + \frac{5 - \sqrt{13}}{6}\right)x + \left(\frac{5 + \sqrt{13}}{6}\right)\left(\frac{5 - \sqrt{13}}{6}\right)\right] =$$

$$3\left[x^2 - \left(\frac{5}{6} + \frac{5}{6}\right)x + \left(\frac{5^2 - (\sqrt{13})^2}{6^2}\right)\right] =$$

Factoring-7

$$3\left[x^2 - \frac{5}{3}x + \left(\frac{25-13}{36}\right)\right] = 3\left[x^2 - \frac{5}{3}x + \frac{12}{36}\right] = 3\left[x^2 - \frac{5}{3}x + \frac{1}{3}\right] =$$

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Factoring General Polynomials

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x	-2	-1	1	2
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The Rational Root Test

Suppose $p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ is a polynomial with *integer* coefficients, and $r = \dfrac{m}{q}$ is a rational number expressed in lowest terms.

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Example 8: Factor the third degree polynomial
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Solution: Suppose $\dfrac{m}{q}$ is a root. Then m divides 2 and q divides 3, so the possibilities are $m = -2, -1, 1, 2$, and $q = -3, -1, 1$, and 3, leaving us with 16 possible values for $\dfrac{m}{q}$, as shown in the following table.

	$m = -2$	$m = -1$	$m = 1$	$m = 2$
$q = -3$	$\frac{m}{q} = \frac{-2}{-3} = \frac{2}{3}$	$\frac{m}{q} = \frac{-1}{-3} = \frac{1}{3}$	$\frac{m}{q} = \frac{1}{-3} = -\frac{1}{3}$	$\frac{m}{q} = \frac{2}{-3} = -\frac{2}{3}$
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$q = 3$	$\frac{m}{q} = \frac{-2}{3} = -\frac{2}{3}$	$\frac{m}{q} = \frac{-1}{3} = -\frac{1}{3}$	$\frac{m}{q} = \frac{1}{3}$	$\frac{m}{q} = \frac{2}{3}$

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\multicolumn{1}{r|}{3x-2} & 3x^3 & -8x^2 & +x & +2 \\
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x	-2	-1	$-\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	1	2
$p(x)$	-56	-4	$-\frac{4}{3}$	$\frac{2}{3}$	$\frac{2}{9}$	0	-2	-4

We see that $\dfrac{2}{3}$ is the only rational root. In particular, $x - \dfrac{2}{3}$ is a factor of $p(x)$. We could perform the long division of $x - \dfrac{2}{3}$ into $3x^3 - 8x^2 + x + 2$, but this would lead to some messy calculations with fractions that can easily be avoided by noticing that $x - \dfrac{2}{3} = \dfrac{1}{3}(3x - 2)$. We divide $3x - 2$ into $p(x)$:

$$
\begin{array}{rrrrr}
 & & x^2 & -2x & -1 \\ \cline{2-5}
3x-2 & \multicolumn{1}{|r}{3x^3} & -8x^2 & +x & +2 \\
 & 3x^3 & -2x^2 & & \\ \cline{2-3}
 & & -6x^2 & +x & \\
 & & -6x^2 & +4x & \\ \cline{3-4}
 & & & -3x & +2 \\
 & & & -3x & +2 \\ \cline{4-5}
 & & & & 0
\end{array}
$$

So now we have $p(x) = 3x^3 - 8x^2 + x + 2 = (3x - 2)(x^2 - 2x - 1) = 3\left(x - \frac{2}{3}\right)(x^2 - 2x - 1)$.

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