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$$3x^2 + 5x + 2 = 3\left[x^2 + \frac{5}{3}x + \frac{2}{3}\right] =$$

Things get even more complicated when the coefficient a of x^2 is not equal to 1: we have to factor it out before proceeding as above:

$$ax^2 + bx + c = a\left[x^2 + \frac{b}{a}x + \frac{c}{a}\right] =$$

$$a\left[x^2 + 2\left(\frac{b}{2a}\right)x + \underbrace{\left(\left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right)}_{0} + \frac{c}{a}\right] =$$

$$a\left[\overbrace{x^2 + 2\left(\frac{b}{2a}\right)x + \left(\frac{b}{2a}\right)^2}^{\left(x+\frac{b}{2a}\right)^2} - \left(\frac{b}{2a}\right)^2 + \frac{c}{a}\right] = a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{4ac}{4a^2}\right] =$$

$$a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2}\right]$$

Example 5:

$$3x^2 + 5x + 2 = 3\left[x^2 + \frac{5}{3}x + \frac{2}{3}\right] =$$

$$3\left[x^2 + 2\left(\frac{5}{6}\right)x + \underbrace{\left(\left(\frac{5}{6}\right)^2 - \left(\frac{5}{6}\right)^2\right)}_{0} + \frac{2}{3}\right] =$$

$$3\left[\overbrace{x^2 + 2\left(\frac{5}{6}\right)x + \left(\frac{5}{6}\right)^2}^{\left(x+\frac{5}{6}\right)^2} - \left(\frac{5}{6}\right)^2 + \frac{2}{3}\left(\frac{12}{12}\right)\right] =$$

$$3\left[\overbrace{x^2+2\left(\frac{5}{6}\right)x+\left(\frac{5}{6}\right)^2}^{\left(x+\frac{5}{6}\right)^2}-\left(\frac{5}{6}\right)^2+\frac{2}{3}\left(\frac{12}{12}\right)\right]=$$

$$3\left[\left(x+\frac{5}{6}\right)^2-\frac{25}{36}+\frac{24}{36}\right]=$$

$$3\left[\overbrace{x^2+2\left(\frac{5}{6}\right)x+\left(\frac{5}{6}\right)^2}^{\left(x+\frac{5}{6}\right)^2}-\left(\frac{5}{6}\right)^2+\frac{2}{3}\left(\frac{12}{12}\right)\right]=$$

$$3\left[\left(x+\frac{5}{6}\right)^2-\frac{25}{36}+\frac{24}{36}\right]=3\left[\left(x+\frac{5}{6}\right)^2-\frac{1}{36}\right]=$$

$$3\left[\overbrace{x^2 + 2\left(\frac{5}{6}\right)x + \left(\frac{5}{6}\right)^2}^{\left(x+\frac{5}{6}\right)^2} - \left(\frac{5}{6}\right)^2 + \frac{2}{3}\left(\frac{12}{12}\right)\right] =$$

$$3\left[\left(x + \frac{5}{6}\right)^2 - \frac{25}{36} + \frac{24}{36}\right] = 3\left[\left(x + \frac{5}{6}\right)^2 - \frac{1}{36}\right] = 3\left[\left(x + \frac{5}{6}\right)^2 - \left(\frac{1}{6}\right)^2\right] =$$

$$3\left[\overbrace{x^2+2\left(\frac{5}{6}\right)x+\left(\frac{5}{6}\right)^2}^{\left(x+\frac{5}{6}\right)^2}-\left(\frac{5}{6}\right)^2+\frac{2}{3}\left(\frac{12}{12}\right)\right]=$$

$$3\left[\left(x+\frac{5}{6}\right)^2-\frac{25}{36}+\frac{24}{36}\right]=3\left[\left(x+\frac{5}{6}\right)^2-\frac{1}{36}\right]=3\left[\left(x+\frac{5}{6}\right)^2-\left(\frac{1}{6}\right)^2\right]=$$

$$3\left(x+\frac{5}{6}+\frac{1}{6}\right)\left(x+\frac{5}{6}-\frac{1}{6}\right)=$$

$$3\left[\overbrace{x^2+2\left(\frac{5}{6}\right)x+\left(\frac{5}{6}\right)^2}^{\left(x+\frac{5}{6}\right)^2}-\left(\frac{5}{6}\right)^2+\frac{2}{3}\left(\frac{12}{12}\right)\right]=$$

$$3\left[\left(x+\frac{5}{6}\right)^2-\frac{25}{36}+\frac{24}{36}\right]=3\left[\left(x+\frac{5}{6}\right)^2-\frac{1}{36}\right]=3\left[\left(x+\frac{5}{6}\right)^2-\left(\frac{1}{6}\right)^2\right]=$$

$$3\left(x+\frac{5}{6}+\frac{1}{6}\right)\left(x+\frac{5}{6}-\frac{1}{6}\right)=3\left(x+1\right)\left(x+\frac{2}{3}\right)=$$

$$3\left[\overbrace{x^2 + 2\left(\frac{5}{6}\right)x + \left(\frac{5}{6}\right)^2}^{\left(x+\frac{5}{6}\right)^2} - \left(\frac{5}{6}\right)^2 + \frac{2}{3}\left(\frac{12}{12}\right)\right] =$$

$$3\left[\left(x + \frac{5}{6}\right)^2 - \frac{25}{36} + \frac{24}{36}\right] = 3\left[\left(x + \frac{5}{6}\right)^2 - \frac{1}{36}\right] = 3\left[\left(x + \frac{5}{6}\right)^2 - \left(\frac{1}{6}\right)^2\right] =$$

$$3\left(x + \frac{5}{6} + \frac{1}{6}\right)\left(x + \frac{5}{6} - \frac{1}{6}\right) = 3\,(x+1)\left(x + \frac{2}{3}\right) = \boxed{(x+1)(3x+2)}$$

We return to the general situation.

$$3\left[\overbrace{x^2+2\left(\frac{5}{6}\right)x+\left(\frac{5}{6}\right)^2}^{\left(x+\frac{5}{6}\right)^2}-\left(\frac{5}{6}\right)^2+\frac{2}{3}\left(\frac{12}{12}\right)\right]=$$

$$3\left[\left(x+\frac{5}{6}\right)^2-\frac{25}{36}+\frac{24}{36}\right]=3\left[\left(x+\frac{5}{6}\right)^2-\frac{1}{36}\right]=3\left[\left(x+\frac{5}{6}\right)^2-\left(\frac{1}{6}\right)^2\right]=$$

$$3\left(x+\frac{5}{6}+\frac{1}{6}\right)\left(x+\frac{5}{6}-\frac{1}{6}\right)=3\,(x+1)\left(x+\frac{2}{3}\right)=\quad (x+1)(3x+2)$$

We return to the general situation. By completing squares, we derived the equation:

$$3\left[\overbrace{x^2+2\left(\frac{5}{6}\right)x+\left(\frac{5}{6}\right)^2}^{\left(x+\frac{5}{6}\right)^2}-\left(\frac{5}{6}\right)^2+\frac{2}{3}\left(\frac{12}{12}\right)\right]=$$

$$3\left[\left(x+\frac{5}{6}\right)^2-\frac{25}{36}+\frac{24}{36}\right]=3\left[\left(x+\frac{5}{6}\right)^2-\frac{1}{36}\right]=3\left[\left(x+\frac{5}{6}\right)^2-\left(\frac{1}{6}\right)^2\right]=$$

$$3\left(x+\frac{5}{6}+\frac{1}{6}\right)\left(x+\frac{5}{6}-\frac{1}{6}\right)=3(x+1)\left(x+\frac{2}{3}\right)=\quad(x+1)(3x+2)$$

We return to the general situation. By completing squares, we derived the equation:

$ax^2+bx+c=$

$$3\left[\overbrace{x^2+2\left(\frac{5}{6}\right)x+\left(\frac{5}{6}\right)^2}^{\left(x+\frac{5}{6}\right)^2}-\left(\frac{5}{6}\right)^2+\frac{2}{3}\left(\frac{12}{12}\right)\right]=$$

$$3\left[\left(x+\frac{5}{6}\right)^2-\frac{25}{36}+\frac{24}{36}\right]=3\left[\left(x+\frac{5}{6}\right)^2-\frac{1}{36}\right]=3\left[\left(x+\frac{5}{6}\right)^2-\left(\frac{1}{6}\right)^2\right]=$$

$$3\left(x+\frac{5}{6}+\frac{1}{6}\right)\left(x+\frac{5}{6}-\frac{1}{6}\right)=3\,(x+1)\left(x+\frac{2}{3}\right)=\quad (x+1)(3x+2)$$

We return to the general situation. By completing squares, we derived the equation:

$$ax^2+bx+c=a\left[\left(x+\frac{b}{2a}\right)^2-\frac{b^2-4ac}{4a^2}\right]$$

We use this to solve the equation $ax^2+bx+c=0$:

$$3\left[\overbrace{x^2+2\left(\frac{5}{6}\right)x+\left(\frac{5}{6}\right)^2}^{\left(x+\frac{5}{6}\right)^2}-\left(\frac{5}{6}\right)^2+\frac{2}{3}\left(\frac{12}{12}\right)\right]=$$

$$3\left[\left(x+\frac{5}{6}\right)^2-\frac{25}{36}+\frac{24}{36}\right]=3\left[\left(x+\frac{5}{6}\right)^2-\frac{1}{36}\right]=3\left[\left(x+\frac{5}{6}\right)^2-\left(\frac{1}{6}\right)^2\right]=$$

$$3\left(x+\frac{5}{6}+\frac{1}{6}\right)\left(x+\frac{5}{6}-\frac{1}{6}\right)=3\,(x+1)\left(x+\frac{2}{3}\right)=\quad (x+1)(3x+2)$$

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$$3\left[\overbrace{x^2+2\left(\frac{5}{6}\right)x+\left(\frac{5}{6}\right)^2}^{\left(x+\frac{5}{6}\right)^2}-\left(\frac{5}{6}\right)^2+\frac{2}{3}\left(\frac{12}{12}\right)\right]=$$

$$3\left[\left(x+\frac{5}{6}\right)^2-\frac{25}{36}+\frac{24}{36}\right]=3\left[\left(x+\frac{5}{6}\right)^2-\frac{1}{36}\right]=3\left[\left(x+\frac{5}{6}\right)^2-\left(\frac{1}{6}\right)^2\right]=$$

$$3\left(x+\frac{5}{6}+\frac{1}{6}\right)\left(x+\frac{5}{6}-\frac{1}{6}\right)=3\,(x+1)\left(x+\frac{2}{3}\right)=\quad (x+1)(3x+2)$$

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$$ax^2+bx+c=0 \qquad\qquad \Longleftrightarrow$$

$$3\left[\overbrace{x^2+2\left(\frac{5}{6}\right)x+\left(\frac{5}{6}\right)^2}^{\left(x+\frac{5}{6}\right)^2}-\left(\frac{5}{6}\right)^2+\frac{2}{3}\left(\frac{12}{12}\right)\right]=$$

$$3\left[\left(x+\frac{5}{6}\right)^2-\frac{25}{36}+\frac{24}{36}\right]=3\left[\left(x+\frac{5}{6}\right)^2-\frac{1}{36}\right]=3\left[\left(x+\frac{5}{6}\right)^2-\left(\frac{1}{6}\right)^2\right]=$$

$$3\left(x+\frac{5}{6}+\frac{1}{6}\right)\left(x+\frac{5}{6}-\frac{1}{6}\right)=3\,(x+1)\left(x+\frac{2}{3}\right)=\quad (x+1)(3x+2)$$

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$$ax^2+bx+c=0 \iff$$

$$\left(x+\frac{b}{2a}\right)^2-\frac{b^2-4ac}{4a^2}=0 \iff$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2} \iff \left(\text{ if } b^2-4ac\geq 0\right)$$

$$3\left[\overbrace{x^2 + 2\left(\frac{5}{6}\right)x + \left(\frac{5}{6}\right)^2}^{\left(x+\frac{5}{6}\right)^2} - \left(\frac{5}{6}\right)^2 + \frac{2}{3}\left(\frac{12}{12}\right)\right] =$$

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$$ax^2 + bx + c = 0 \iff$$

$$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2} = 0 \iff$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2} \iff \left(\text{ if } b^2 - 4ac \geq 0\right)$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} = \pm\frac{\sqrt{b^2 - 4ac}}{2a} \iff$$

$$3\left[\overbrace{x^2+2\left(\frac{5}{6}\right)x+\left(\frac{5}{6}\right)^2}^{\left(x+\frac{5}{6}\right)^2}-\left(\frac{5}{6}\right)^2+\frac{2}{3}\left(\frac{12}{12}\right)\right]=$$

$$3\left[\left(x+\frac{5}{6}\right)^2-\frac{25}{36}+\frac{24}{36}\right]=3\left[\left(x+\frac{5}{6}\right)^2-\frac{1}{36}\right]=3\left[\left(x+\frac{5}{6}\right)^2-\left(\frac{1}{6}\right)^2\right]=$$

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$$\left(x+\frac{b}{2a}\right)^2-\frac{b^2-4ac}{4a^2}=0 \iff$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2} \iff \left(\text{ if } b^2-4ac\geq 0\right)$$

$$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}=\pm\frac{\sqrt{b^2-4ac}}{2a} \iff$$

$$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}=$$

$$3\left[\overbrace{x^2+2\left(\frac{5}{6}\right)x+\left(\frac{5}{6}\right)^2}^{\left(x+\frac{5}{6}\right)^2}-\left(\frac{5}{6}\right)^2+\frac{2}{3}\left(\frac{12}{12}\right)\right]=$$

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$$ax^2+bx+c=0 \iff$$

$$\left(x+\frac{b}{2a}\right)^2-\frac{b^2-4ac}{4a^2}=0 \iff$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2} \iff \left(\text{ if } b^2-4ac\geq 0\right)$$

$$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}=\pm\frac{\sqrt{b^2-4ac}}{2a} \iff$$

$$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}=\frac{-b\pm\sqrt{b^2-4ac}}{2a},$$

$$3\left[\overbrace{x^2+2\left(\frac{5}{6}\right)x+\left(\frac{5}{6}\right)^2}^{\left(x+\frac{5}{6}\right)^2}-\left(\frac{5}{6}\right)^2+\frac{2}{3}\left(\frac{12}{12}\right)\right]=$$

$$3\left[\left(x+\frac{5}{6}\right)^2-\frac{25}{36}+\frac{24}{36}\right]=3\left[\left(x+\frac{5}{6}\right)^2-\frac{1}{36}\right]=3\left[\left(x+\frac{5}{6}\right)^2-\left(\frac{1}{6}\right)^2\right]=$$

$$3\left(x+\frac{5}{6}+\frac{1}{6}\right)\left(x+\frac{5}{6}-\frac{1}{6}\right)=3\,(x+1)\left(x+\frac{2}{3}\right)=\boxed{(x+1)(3x+2)}$$

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$$ax^2+bx+c=0 \iff$$

$$\left(x+\frac{b}{2a}\right)^2-\frac{b^2-4ac}{4a^2}=0 \iff$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2} \iff \left(\text{ if } b^2-4ac\geq 0\right)$$

$$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}=\pm\frac{\sqrt{b^2-4ac}}{2a} \iff$$

$x=-\dfrac{b}{2a}\pm\dfrac{\sqrt{b^2-4ac}}{2a}=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$, which is the familiar Quadratic Formula for the roots of the general quadratic equation.