Exercises with Polynomials

Exercise Set I.

Multiply the following expressions.

(I.1) $\left(\frac{2}{3}x + 4y\right)\left(\frac{1}{2}x - 3y + 1\right)$ Solution

(I.2) $(2x + a)(ax - 1)(2x - a)$ Solution

(I.3) $(2x + 1)^4$ Solution

(I.4) $(2x - 1)^4$ Solution

(I.5) $(3x + 2)^4$ Solution

(I.6) $\left(x^2 + 2\right)^4$ Solution

Exercise Set II.

Divide the following expressions.

(II.1) $\dfrac{2x^3 - x^2 - 3x + 14}{x + 2}$ Solution

(II.2) $\dfrac{x^3 + \frac{7}{8}}{2x + 1}$ Solution

(II.3) $\dfrac{2x^4 + x^3 + 1}{x^2 - 2}$ Solution

(II.4) $\dfrac{x^3 + x^2 - 13x + 3}{x - 3}$ Solution

(II.5) $\dfrac{x^4 + x^3 - 9x^2 + x + 2}{x^2 + 3x - 2}$ Solution

Solution Set I.

Back to Questions

(I.1)

$$\left(\frac{2}{3}x + 4y\right)\left(\frac{1}{2}x - 3y + 1\right)$$

Solution: $\left(\frac{2}{3}x + 4y\right)\left(\frac{1}{2}x - 3y + 1\right) =$

Solution Set I.

Back to Questions

(I.1) $\left(\frac{2}{3}x + 4y\right)\left(\frac{1}{2}x - 3y + 1\right)$

Solution: $\left(\frac{2}{3}x + 4y\right)\left(\frac{1}{2}x - 3y + 1\right) =$

$\frac{2}{3}x\left(\frac{1}{2}x - 3y + 1\right) + 4y\left(\frac{1}{2}x - 3y + 1\right) =$

Solution Set I.

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(I.1) $\left(\frac{2}{3}x + 4y\right)\left(\frac{1}{2}x - 3y + 1\right)$

Solution: $\left(\frac{2}{3}x + 4y\right)\left(\frac{1}{2}x - 3y + 1\right) =$

$$\frac{2}{3}x\left(\frac{1}{2}x - 3y + 1\right) + 4y\left(\frac{1}{2}x - 3y + 1\right) =$$

$$\frac{2}{3}x\frac{1}{2}x + \frac{2}{3}x(-3y) + \frac{2}{3}x(1) + 4y\frac{1}{2}x + 4y(-3y) + 4y(1) =$$

Solution Set I.

Back to Questions

(I.1) $\left(\frac{2}{3}x + 4y\right)\left(\frac{1}{2}x - 3y + 1\right)$

Solution: $\left(\frac{2}{3}x + 4y\right)\left(\frac{1}{2}x - 3y + 1\right) =$

$$\frac{2}{3}x\left(\frac{1}{2}x - 3y + 1\right) + 4y\left(\frac{1}{2}x - 3y + 1\right) =$$

$$\frac{2}{3}x\frac{1}{2}x + \frac{2}{3}x(-3y) + \frac{2}{3}x(1) + 4y\frac{1}{2}x + 4y(-3y) + 4y(1) =$$

$$\frac{1}{3}x^2 - 2xy + \frac{2}{3}x + 2xy - 12y^2 + 4y =$$

Solution Set I.

Back to Questions

(I.1) $\left(\frac{2}{3}x+4y\right)\left(\frac{1}{2}x-3y+1\right)$

Solution: $\left(\frac{2}{3}x+4y\right)\left(\frac{1}{2}x-3y+1\right)=$

$\frac{2}{3}x\left(\frac{1}{2}x-3y+1\right)+4y\left(\frac{1}{2}x-3y+1\right)=$

$\frac{2}{3}x\frac{1}{2}x+\frac{2}{3}x(-3y)+\frac{2}{3}x(1)+4y\frac{1}{2}x+4y(-3y)+4y(1)=$

$\frac{1}{3}x^2-2xy+\frac{2}{3}x+2xy-12y^2+4y=$

$\frac{1}{3}x^2-12y^2+\frac{2}{3}x+4y$

Back to Questions

(I.2) $(2x + a)(ax - 1)(2x - a)$

Solution: $(2x + a)(ax - 1)(2x - a) =$

Back to Questions

(I.2) $(2x+a)(ax-1)(2x-a)$

Solution: $(2x+a)(ax-1)(2x-a)=$

$(2x+a)(2x-a)(ax-1)=$

Back to Questions

(I.2) $(2x + a)(ax - 1)(2x - a)$

Solution: $(2x + a)(ax - 1)(2x - a) =$

$(2x + a)(2x - a)(ax - 1) =$

$(4x^2 - a^2)(ax - 1) =$

Back to Questions

(I.2) $(2x+a)(ax-1)(2x-a)$

Solution: $(2x+a)(ax-1)(2x-a) =$

$(2x+a)(2x-a)(ax-1) =$

$(4x^2-a^2)(ax-1) =$

$(4x^2-a^2)(ax)+(4x^2-a^2)(-1) =$

Back to Questions

(I.2) $(2x + a)(ax - 1)(2x - a)$

Solution: $(2x + a)(ax - 1)(2x - a) =$

$(2x + a)(2x - a)(ax - 1) =$

$(4x^2 - a^2)(ax - 1) =$

$(4x^2 - a^2)(ax) + (4x^2 - a^2)(-1) =$

$4ax^3 - a^3x - 4x^2 + a^2 =$

Back to Questions

(I.2) $(2x + a)(ax - 1)(2x - a)$

Solution: $(2x + a)(ax - 1)(2x - a) =$

$(2x + a)(2x - a)(ax - 1) =$

$(4x^2 - a^2)(ax - 1) =$

$(4x^2 - a^2)(ax) + (4x^2 - a^2)(-1) =$

$4ax^3 - a^3x - 4x^2 + a^2 =$ $4ax^3 - 4x^2 - a^3x + a^2$

(I.3) $(2x+1)^4$ Back to Questions

Solution: $(2x+1)^4 =$

(I.3) $(2x+1)^4$

Back to Questions

Solution: $(2x+1)^4 = \left[(2x+1)^2\right]^2 =$

(I.3) $(2x+1)^4$

Back to Questions

Solution: $(2x+1)^4 = \left[(2x+1)^2\right]^2 = \left[(2x)^2 + 2(2x)(1) + 1^2\right]^2 =$

(I.3) $(2x+1)^4$

Back to Questions

Solution: $(2x+1)^4 = \left[(2x+1)^2\right]^2 = \left[(2x)^2 + 2(2x)(1) + 1^2\right]^2 =$

$\left[4x^2 + 4x + 1\right]^2 =$

(I.3) $(2x+1)^4$

Back to Questions

Solution: $(2x+1)^4 = \left[(2x+1)^2\right]^2 = \left[(2x)^2 + 2(2x)(1) + 1^2\right]^2 =$

$\left[4x^2 + 4x + 1\right]^2 =$ $16x^4 + 32x^3 + 24x^2 + 8x + 1$, which we now compute using long multiplication:

			$4x^2$	$+4x$	$+1$
			$4x^2$	$+4x$	$+1$
			$4x^2$	$+4x$	$+1$
		$16x^3$	$+16x^2$	$+4x$	
	$16x^4$	$+16x^3$	$+4x^2$		
$16x^4$	$+32x^3$	$+24x^2$	$+8x$	$+1$	

Better Solution: Since we know that

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4,$$

we let $a =$

Better Solution: Since we know that

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4,$$

we let $a = 2x$ and $b = 1$ to get

$(2x+1)^4 =$

Better Solution: Since we know that

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4,$$

we let $a = 2x$ and $b = 1$ to get

$(2x+1)^4 =$

$(2x)^4 + 4(2x)^3(1) + 6(2x)^2(1)^2 + 4(2x)(1)^3 + 1^4 =$

Better Solution: Since we know that

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4,$$

we let $a = 2x$ and $b = 1$ to get

$(2x+1)^4 =$

$(2x)^4 + 4(2x)^3(1) + 6(2x)^2(1)^2 + 4(2x)(1)^3 + 1^4 =$

$16x^4 + 32x^3 + 24x^2 + 8x + 1$

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(I.4) $(2x-1)^4$ Solution

Solution: Since we know that

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4,$$

we let $a =$

Back to Questions

(I.4)

$(2x-1)^4$ Solution

Solution: Since we know that

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4,$$

we let $a = 2x$ and $b = -1$ to get

$(2x-1)^4 =$

Back to Questions

(I.4) $(2x - 1)^4$ Solution

Solution: Since we know that

$$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4,$$

we let $a = 2x$ and $b = -1$ to get

$(2x - 1)^4 =$

$(2x)^4 + 4(2x)^3(-1) + 6(2x)^2(-1)^2 + 4(2x)(-1)^3 + (-1)^4 =$

Back to Questions

(I.4) $(2x-1)^4$ Solution

Solution: Since we know that

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4,$$

we let $a = 2x$ and $b = -1$ to get

$(2x-1)^4 =$

$(2x)^4 + 4(2x)^3(-1) + 6(2x)^2(-1)^2 + 4(2x)(-1)^3 + (-1)^4 =$

$16x^4 - 32x^3 + 24x^2 - 8x + 1$

Back to Questions

(I.5) $(3x+2)^4$ Solution

Solution: Since we know that

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4,$$

we let $a =$

Back to Questions

(I.5) $(3x+2)^4$ Solution

Solution: Since we know that

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4,$$

we let $a = 3x$ and $b = 2$ to get

$(3x+2)^4 =$

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(I.5) $(3x+2)^4$ Solution

Solution: Since we know that

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4,$$

we let $a = 3x$ and $b = 2$ to get

$(3x+2)^4 =$

$(3x)^4 + 4(3x)^3(2) + 6(3x)^2(2)^2 + 4(3x)(2)^3 + (2)^4 =$

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(I.5)

$(3x+2)^4$ Solution

Solution: Since we know that

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4,$$

we let $a = 3x$ and $b = 2$ to get

$(3x+2)^4 =$

$(3x)^4 + 4(3x)^3(2) + 6(3x)^2(2)^2 + 4(3x)(2)^3 + (2)^4 =$

$81x^4 + 216x^3 + 216x^2 + 96x + 16$

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(I.6) $\left(x^2 + 2\right)^4$ Solution

Solution: Since we know that

$$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4,$$

we let $a =$

Back to Questions

(I.6) $\left(x^2+2\right)^4$ Solution

Solution: Since we know that

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4,$$

we let $a = x^2$ and $b = 2$ to get

$(x^2+2)^4 =$

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(I.6) $\left(x^2+2\right)^4$ Solution

Solution: Since we know that

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4,$$

we let $a = x^2$ and $b = 2$ to get

$(x^2+2)^4 =$

$(x^2)^4 + 4(x^2)^3(2) + 6(x^2)^2(2)^2 + 4(x^2)(2)^3 + (2)^4 =$

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(I.6) $\left(x^2+2\right)^4$ Solution

Solution: Since we know that

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4,$$

we let $a = x^2$ and $b = 2$ to get

$(x^2+2)^4 =$

$(x^2)^4 + 4(x^2)^3(2) + 6(x^2)^2(2)^2 + 4(x^2)(2)^3 + (2)^4 =$

$x^8 + 8x^6 + 24x^4 + 32x^2 + 16$

Solution Set II.

Divide the following expressions.

(II.1) $\dfrac{2x^3 - x^2 - 3x + 14}{x + 2}$

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$$
\begin{array}{r|rrrr}
 & 2x^2 & -5x & +7 & \\
\hline
x+2 & 2x^3 & -x^2 & -3x & +14 \\
 & 2x^3 & +4x^2 & & \\
\hline
 & & -5x^2 & -3x & +14 \\
 & & -5x^2 & -10x & \\
\hline
 & & & 7x & +14 \\
 & & & 7x & +14 \\
\hline
 & & & & 0
\end{array}
$$

Solution:

so $\dfrac{2x^3 - x^2 - 3x + 14}{x + 2} =$

Solution Set II.

Divide the following expressions.

(II.1) $\dfrac{2x^3 - x^2 - 3x + 14}{x + 2}$

Back to Questions

Solution:

$$
\begin{array}{r|rrrr}
 & & 2x^2 & -5x & +7 \\
\hline
x+2 & 2x^3 & -x^2 & -3x & +14 \\
 & 2x^3 & +4x^2 & & \\
\hline
 & & -5x^2 & -3x & +14 \\
 & & -5x^2 & -10x & \\
\hline
 & & & 7x & +14 \\
 & & & 7x & +14 \\
\hline
 & & & & 0
\end{array}
$$

so $\dfrac{2x^3 - x^2 - 3x + 14}{x + 2} = 2x^2 - 5x + 7$

(II.2) $\dfrac{x^3+\frac{7}{8}}{2x+1} = \dfrac{1}{8}\left[\dfrac{8x^3+7}{2x+1}\right]$

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Solution:

$$
\begin{array}{r|rrrr}
 & 4x^2 & -2x & +1 & \\
\hline
2x+1 & 8x^3 & +0x^2 & +0x & +7 \\
 & 8x^3 & +4x^2 & & \\
\hline
 & & -4x^2 & & +7 \\
 & & -4x^2 & -2x & \\
\hline
 & & & 2x & +7 \\
 & & & 2x & +1 \\
\hline
 & & & & 6
\end{array}
$$

so $\dfrac{x^3+\frac{7}{8}}{2x+1} = \dfrac{1}{8}\left[\dfrac{8x^3+7}{2x+1}\right] =$

(II.2) $\frac{x^3+\frac{7}{8}}{2x+1} = \frac{1}{8}\left[\frac{8x^3+7}{2x+1}\right]$

Back to Questions

$$
\begin{array}{r|rrrr}
 & 4x^2 & -2x & +1 & \\
\hline
2x+1 & 8x^3 & +0x^2 & +0x & +7 \\
 & 8x^3 & +4x^2 & & \\
\hline
 & & -4x^2 & & +7 \\
 & & -4x^2 & -2x & \\
\hline
 & & & 2x & +7 \\
 & & & 2x & +1 \\
\hline
 & & & & 6
\end{array}
$$

Solution:

so $\frac{x^3+\frac{7}{8}}{2x+1} = \frac{1}{8}\left[\frac{8x^3+7}{2x+1}\right] = \frac{1}{8}\left[4x^2-2x+1+\frac{6}{2x+1}\right]$

(II.3) $\dfrac{2x^4 + x^3 + 1}{x^2 - 2}$

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Solution:

$$
\begin{array}{r|rrrrr}
 & & & 2x^2 & +x & +4 \\
\hline
x^2 - 2 & 2x^4 & +x^3 & +0x^2 & +0x & +1 \\
 & 2x^4 & & -4x^2 & & \\
\hline
 & & x^3 & +4x^2 & & +1 \\
 & & x^3 & & -2x & \\
\hline
 & & & 4x^2 & +2x & +1 \\
 & & & 4x^2 & & -8 \\
\hline
 & & & & 2x & +9
\end{array}
$$

so $\dfrac{2x^4 + x^3 + 1}{x^2 - 2} =$

(II.3) $\dfrac{2x^4 + x^3 + 1}{x^2 - 2}$

Back to Questions

Solution:

$$
\begin{array}{r|rrrrr}
 & & & 2x^2 & +x & +4 \\
\hline
x^2 - 2 & 2x^4 & +x^3 & +0x^2 & +0x & +1 \\
 & 2x^4 & & -4x^2 & & \\
\hline
 & & x^3 & +4x^2 & & +1 \\
 & & x^3 & & -2x & \\
\hline
 & & & 4x^2 & +2x & +1 \\
 & & & 4x^2 & & -8 \\
\hline
 & & & & 2x & +9
\end{array}
$$

so $\dfrac{2x^4 + x^3 + 1}{x^2 - 2} = 2x^2 + x + 4 + \dfrac{2x + 9}{x^2 - 2}$

(II.4) $\dfrac{x^3+x^2-13x+3}{x-3}$

Back to Questions

Solution:

$$
\begin{array}{r|rrrr}
 & x^2 & +4x & -1 & \\
\hline
x-3 & x^3 & +x^2 & -13x & +3 \\
 & x^3 & -3x^2 & & \\
\hline
 & & +4x^2 & -13x & +3 \\
 & & 4x^2 & -12x & \\
\hline
 & & & -x & +3 \\
 & & & -x & +3 \\
\hline
\end{array}
$$

so $\dfrac{x^3+x^2-13x+3}{x-3} =$

(II.4) $\dfrac{x^3 + x^2 - 13x + 3}{x - 3}$

Back to Questions

Solution:

$$
\begin{array}{r|rrrr}
 & x^2 & +4x & -1 & \\
\hline
x - 3 & x^3 & +x^2 & -13x & +3 \\
 & x^3 & -3x^2 & & \\
\hline
 & & +4x^2 & -13x & +3 \\
 & & 4x^2 & -12x & \\
\hline
 & & & -x & +3 \\
 & & & -x & +3 \\
\hline
\end{array}
$$

so $\dfrac{x^3 + x^2 - 13x + 3}{x - 3} = x^2 + 4x - 1$

(II.5) $\dfrac{x^4 + x^3 - 9x^2 + x + 2}{x^2 + 3x - 2}$

Back to Questions

Solution:

$$
\begin{array}{r|rrrrr}
 & & & x^2 & -2x & -1 \\
\hline
x^2 + 3x - 2 & x^4 & +x^3 & -9x^2 & +x & +2 \\
 & x^4 & +3x^3 & -2x^2 & & \\
\hline
 & & -2x^3 & -7x^2 & +x & +2 \\
 & & -2x^3 & -6x^2 & +4x & \\
\hline
 & & & -x^2 & -3x & +2 \\
 & & & -x^2 & -3x & +2 \\
\hline
\end{array}
$$

so $\dfrac{x^4 + x^3 - 9x^2 + x + 2}{x^2 + 3x - 2} =$

(II.5) $\dfrac{x^4 + x^3 - 9x^2 + x + 2}{x^2 + 3x - 2}$

Back to Questions

Solution:

$$
\begin{array}{r|rrrrr}
 & & & x^2 & -2x & -1 \\
\hline
x^2 + 3x - 2 & x^4 & +x^3 & -9x^2 & +x & +2 \\
 & x^4 & +3x^3 & -2x^2 & & \\
\hline
 & & -2x^3 & -7x^2 & +x & +2 \\
 & & -2x^3 & -6x^2 & +4x & \\
\hline
 & & & -x^2 & -3x & +2 \\
 & & & -x^2 & -3x & +2 \\
\hline
\end{array}
$$

so $\dfrac{x^4 + x^3 - 9x^2 + x + 2}{x^2 + 3x - 2} = x^2 - 2x - 1$