Exercise Set I. Factor the following polynomials.

Exit Acrobat

(I.1) $3x^2 + 11x - 4$ Solution

(I.2) $2x^2 + 2x - 1$ Solution

(I.3) $2x^2 + 2x + 1$ Solution

(I.4) $x^4 - 8x$ Solution

(I.5) $2x^3 - 4x^2 + 3x - 1$ Solution

(I.6) $10x^4 + 41x^3 + 12x^2 - 7x - 2$ Solution

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Solution Set I.

(I.1) $3x^2 + 11x - 4$

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so $3x^2 + 11x - 4 = 3(x - (-4))\left(x - \dfrac{1}{3}\right) =$

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so $3x^2 + 11x - 4 = 3(x - (-4))\left(x - \dfrac{1}{3}\right) = (x+4)(3x-1).$

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The possible values of a are $-4, -2, -1, 1, 2, 4$ with the corresponding values of b being $1, 2, 4, -4, -2, -1$ respectively. We construct a table:

a	b	$a + 3b$
-4	1	-1
-2	2	4
-1	4	

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a	b	$a + 3b$
-4	1	-1
-2	2	4
-1	4	11
1	-4	-11
2	-2	-4
4	-1	1

so we take $a = -1$ and $b = 4$

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(I.2) $2x^2 + 2x - 1$

Solution: $2x^2 + 2x - 1$ must be of the form

$(2x + a)(x + b) = 2x^2 + (a + 2b)x + ab$

so we must have $a + 2b = 2$ and $ab = -1$.

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The possible values of a are -1 and 1 with the corresponding values of b being 1 and -1 respectively. We construct a table:

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so there are no integer values of a and b that satisfy the requirements.

We resort to the Quadratic Formula: the roots of $2x^2 + 2x - 1$ are

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$$a(x - r_1)(x - r_2) = \boxed{2\left(x - \frac{-1 + \sqrt{3}}{2}\right)\left(x - \frac{-1 - \sqrt{3}}{2}\right)}$$

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(I.3) $2x^2 + 2x + 1$

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The required factorization therefore involves complex numbers, which are beyond the scope of this course. The complex factorization is:

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The required factorization therefore involves complex numbers, which are beyond the scope of this course. The complex factorization is:

$$a(x - r_1)(x - r_2) = 2\left(x - \frac{-1 + i}{2}\right)\left(x - \frac{-1 - i}{2}\right)$$

Note: The ability to calculate with complex numbers is not required in this course: we only need to know when a polynomial has compex roots, so that we can treat it appropriately. Among other things, we know that we should not even try to factor quadratics with complex roots.

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(I.4) $x^4 - 8x$

Solution: $x^4 - 8x =$

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(I.4) $x^4 - 8x$

Solution: $x^4 - 8x = x(x^3 - 2^3) = x(x-2)(x^2+2x+4)$

(I.5) $2x^3 - 4x^2 + 3x - 1$

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(I.5) $2x^3 - 4x^2 + 3x - 1$

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Solution: Since 1 and -1 are the only possible integer roots of the polynomial, we test them first:

$2(1)^3 - 4(1)^2 + 3(1) - 1 = 0,$

(I.5) $2x^3 - 4x^2 + 3x - 1$

Back to Questions

Solution: Since 1 and -1 are the only possible integer roots of the polynomial, we test them first: $2(1)^3 - 4(1)^2 + 3(1) - 1 = 0$, so we know that $x - 1$ is a factor of the polynomial. We divide:

$$
\begin{array}{r|rrrr}
 & 2x^2 & -2x & +1 & \\
\hline
x-1 & 2x^3 & -4x^2 & +3x & -1 \\
 & 2x^3 & -2x^2 & & \\
\hline
 & & -2x^2 & +3x & -1 \\
 & & -2x^2 & +2x & \\
\hline
 & & & x & -1 \\
 & & & x & -1 \\
\hline
 & & & & 0
\end{array}
$$

(I.5) $2x^3 - 4x^2 + 3x - 1$

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\end{array}
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Thus we have $2x^3 - 4x^2 + 3x - 1 =$

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Thus we have $2x^3 - 4x^2 + 3x - 1 = (x - 1)(2x^2 - 2x + 1)$

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 & & -2x^2 & +2x & \\
\hline
 & & & x & -1 \\
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\hline
 & & & & 0
\end{array}
$$

Thus we have $2x^3 - 4x^2 + 3x - 1 =$ $(x-1)(2x^2 - 2x + 1)$
(Since the roots of $2x^2 - 2x + 1$ are imaginary, we may leave it unfactored.)

(I.6) $p(x) = 10x^4 + 41x^3 + 12x^2 - 7x - 2$

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(I.6) $p(x) = 10x^4 + 41x^3 + 12x^2 - 7x - 2$

Solution: By the Rational Root Test, if $\frac{m}{q}$ is a root of the polynomial, its denominator q must be

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(I.6) $p(x) = 10x^4 + 41x^3 + 12x^2 - 7x - 2$

Solution: By the Rational Root Test, if $\frac{m}{q}$ is a root of the polynomial, its denominator q must be 1, 2, 5, or 10, and its numerator m must be

Back to Questions

(I.6) $p(x) = 10x^4 + 41x^3 + 12x^2 - 7x - 2$

Solution: By the Rational Root Test, if $\frac{m}{q}$ is a root of the polynomial, its denominator q must be 1, 2, 5, or 10, and its numerator m must be $-2, -1, 1,$ or 2 so there are at most 16 possibilities to be checked.

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	$m = -2$	$m = -1$	$m = 1$	$m = 2$
$q = 1$	$\frac{-2}{1} = -2$	$\frac{-1}{1} = -1$	$\frac{1}{1}$	$\frac{2}{1} = -2$
$q = 2$	$\frac{-2}{2} = -1$	$\frac{-1}{2} = -\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{2} = 1$
$q = 5$	$\frac{-2}{5} = -\frac{2}{5}$	$\frac{-1}{5} = -\frac{1}{5}$	$\frac{1}{5}$	$\frac{2}{5}$
$q = 10$	$\frac{-2}{10} = -\frac{1}{5}$	$\frac{-1}{10}$	$\frac{1}{10}$	$\frac{2}{10} = \frac{1}{5}$

Of these, the only integer possibilities are -2, -1, 1, and 2. We check them first:

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	$m = -2$	$m = -1$	$m = 1$	$m = 2$
$q = 1$	$\frac{-2}{1} = -2$	$\frac{-1}{1} = -1$	$\frac{1}{1}$	$\frac{2}{1} = -2$
$q = 2$	$\frac{-2}{2} = -1$	$\frac{-1}{2} = -\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{2} = 1$
$q = 5$	$\frac{-2}{5} = -\frac{2}{5}$	$\frac{-1}{5} = -\frac{1}{5}$	$\frac{1}{5}$	$\frac{2}{5}$
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Of these, the only integer possibilities are -2, -1, 1, and 2. We check them first:

Letting $x = -2, -1, 1, 2$, we have

(I.6) $p(x) = 10x^4 + 41x^3 + 12x^2 - 7x - 2$

Solution: By the Rational Root Test, if $\frac{m}{q}$ is a root of the polynomial, its denominator q must be 1, 2, 5, or 10, and its numerator m must be $-2, -1, 1,$ or 2 so there are at most 16 possibilities to be checked.

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$q = 5$	$\frac{-2}{5} = -\frac{2}{5}$	$\frac{-1}{5} = -\frac{1}{5}$	$\frac{1}{5}$	$\frac{2}{5}$
$q = 10$	$\frac{-2}{10} = -\frac{1}{5}$	$\frac{-1}{10}$	$\frac{1}{10}$	$\frac{2}{10} = \frac{1}{5}$

Of these, the only integer possibilities are -2, -1, 1, and 2. We check them first:

Letting $x = -2, -1, 1, 2$, we have

$p(-2) = 10(-2)^4 + 41(-2)^3 + 12(-2)^2 - 7(-2) - 2 = 160 - 328 + 48 + 14 - 2 = -108,$

(I.6) $p(x) = 10x^4 + 41x^3 + 12x^2 - 7x - 2$

Solution: By the Rational Root Test, if $\frac{m}{q}$ is a root of the polynomial, its denominator q must be 1, 2, 5, or 10, and its numerator m must be $-2, -1, 1,$ or 2 so there are at most 16 possibilities to be checked.

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$q = 1$	$\frac{-2}{1} = -2$	$\frac{-1}{1} = -1$	$\frac{1}{1}$	$\frac{2}{1} = -2$
$q = 2$	$\frac{-2}{2} = -1$	$\frac{-1}{2} = -\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{2} = 1$
$q = 5$	$\frac{-2}{5} = -\frac{2}{5}$	$\frac{-1}{5} = -\frac{1}{5}$	$\frac{1}{5}$	$\frac{2}{5}$
$q = 10$	$\frac{-2}{10} = -\frac{1}{5}$	$\frac{-1}{10}$	$\frac{1}{10}$	$\frac{2}{10} = \frac{1}{5}$

Of these, the only integer possibilities are -2, -1, 1, and 2. We check them first:

Letting $x = -2, -1, 1, 2$, we have

$$p(-2) = 10(-2)^4 + 41(-2)^3 + 12(-2)^2 - 7(-2) - 2 = 160 - 328 + 48 + 14 - 2 = -108,$$
$$p(-1) == 10(-1)^4 + 41(-1)^3 + 12(-1)^2 - 7(-1) - 2 = 10 - 41 + 12 + 7 - 2 = -14,$$

(I.6) $p(x) = 10x^4 + 41x^3 + 12x^2 - 7x - 2$

Solution: By the Rational Root Test, if $\dfrac{m}{q}$ is a root of the polynomial, its denominator q must be 1, 2, 5, or 10, and its numerator m must be $-2, -1, 1$, or 2 so there are at most 16 possibilities to be checked.

Back to Questions

	$m = -2$	$m = -1$	$m = 1$	$m = 2$
$q = 1$	$\frac{-2}{1} = -2$	$\frac{-1}{1} = -1$	$\frac{1}{1}$	$\frac{2}{1} = -2$
$q = 2$	$\frac{-2}{2} = -1$	$\frac{-1}{2} = -\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{2} = 1$
$q = 5$	$\frac{-2}{5} = -\frac{2}{5}$	$\frac{-1}{5} = -\frac{1}{5}$	$\frac{1}{5}$	$\frac{2}{5}$
$q = 10$	$\frac{-2}{10} = -\frac{1}{5}$	$\frac{-1}{10}$	$\frac{1}{10}$	$\frac{2}{10} = \frac{1}{5}$

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$p(1) = 10(1)^4 + 41(1)^3 + 12(1)^2 - 7(1) - 2 = 10 + 41 + 12 - 7 - 2 = 54,$

(I.6) $p(x) = 10x^4 + 41x^3 + 12x^2 - 7x - 2$

Solution: By the Rational Root Test, if $\frac{m}{q}$ is a root of the polynomial, its denominator q must be 1, 2, 5, or 10, and its numerator m must be $-2, -1, 1$, or 2 so there are at most 16 possibilities to be checked.

Back to Questions

	$m = -2$	$m = -1$	$m = 1$	$m = 2$
$q = 1$	$\frac{-2}{1} = -2$	$\frac{-1}{1} = -1$	$\frac{1}{1}$	$\frac{2}{1} = -2$
$q = 2$	$\frac{-2}{2} = -1$	$\frac{-1}{2} = -\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{2} = 1$
$q = 5$	$\frac{-2}{5} = -\frac{2}{5}$	$\frac{-1}{5} = -\frac{1}{5}$	$\frac{1}{5}$	$\frac{2}{5}$
$q = 10$	$\frac{-2}{10} = -\frac{1}{5}$	$\frac{-1}{10}$	$\frac{1}{10}$	$\frac{2}{10} = \frac{1}{5}$

Of these, the only integer possibilities are -2, -1, 1, and 2. We check them first:

Letting $x = -2, -1, 1, 2$, we have

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$$p(1) = 10(1)^4 + 41(1)^3 + 12(1)^2 - 7(1) - 2 = 10 + 41 + 12 - 7 - 2 = 54,$$
$$p(2) = 10(2)^4 + 41(2)^3 + 12(2)^2 - 7(2) - 2 = 160 + 328 + 48 - 14 - 2 = 520,$$

so there are no integer roots. We notice that there must be a root between -1 and 1: indeed, since $p(0) = -2$, we know there must be a root between 0 and 1. There are 8 remaining possibilities, shown in the following table:

(I.6) $p(x) = 10x^4 + 41x^3 + 12x^2 - 7x - 2$

Solution: By the Rational Root Test, if $\dfrac{m}{q}$ is a root of the polynomial, its denominator q must be 1, 2, 5, or 10, and its numerator m must be $-2, -1, 1$, or 2 so there are at most 16 possibilities to be checked.

Back to Questions

	$m = -2$	$m = -1$	$m = 1$	$m = 2$
$q = 1$	$\frac{-2}{1} = -2$	$\frac{-1}{1} = -1$	$\frac{1}{1}$	$\frac{2}{1} = -2$
$q = 2$	$\frac{-2}{2} = -1$	$\frac{-1}{2} = -\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{2} = 1$
$q = 5$	$\frac{-2}{5} = -\frac{2}{5}$	$\frac{-1}{5} = -\frac{1}{5}$	$\frac{1}{5}$	$\frac{2}{5}$
$q = 10$	$\frac{-2}{10} = -\frac{1}{5}$	$\frac{-1}{10}$	$\frac{1}{10}$	$\frac{2}{10} = \frac{1}{5}$

Of these, the only integer possibilities are -2, -1, 1, and 2. We check them first:

Letting $x = -2, -1, 1, 2$, we have

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$$p(2) = 10(2)^4 + 41(2)^3 + 12(2)^2 - 7(2) - 2 = 160 + 328 + 48 - 14 - 2 = 520,$$

so there are no integer roots. We notice that there must be a root between -1 and 1: indeed, since $p(0) = -2$, we know there must be a root between 0 and 1. There are 8 remaining possibilities, shown in the following table:

x	$-\frac{1}{2}$	$-\frac{2}{5}$	$-\frac{1}{5}$	$-\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{2}$

We begin evaluating $p(x)$ at possible roots which have the smallest denominators, so as to keep our fractional arithmetic as simple as possible:

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$$p\left(\frac{1}{2}\right) = 10\left(\frac{1}{2}\right)^4 + 41\left(\frac{1}{2}\right)^3 + 12\left(\frac{1}{2}\right)^2 - 7\left(\frac{1}{2}\right) - 2 =$$

We begin evaluating $p(x)$ at possible roots which have the smallest denominators, so as to keep our fractional arithmetic as simple as possible:

$$p\left(\frac{1}{2}\right) = 10\left(\frac{1}{2}\right)^4 + 41\left(\frac{1}{2}\right)^3 + 12\left(\frac{1}{2}\right)^2 - 7\left(\frac{1}{2}\right) - 2 = 10\left(\frac{1}{16}\right) + 41\left(\frac{1}{8}\right) + 12\left(\frac{1}{4}\right) - 7\left(\frac{1}{2}\right) - 2 =$$

We begin evaluating $p(x)$ at possible roots which have the smallest denominators, so as to keep our fractional arithmetic as simple as possible:

$$p\left(\frac{1}{2}\right) = 10\left(\frac{1}{2}\right)^4 + 41\left(\frac{1}{2}\right)^3 + 12\left(\frac{1}{2}\right)^2 - 7\left(\frac{1}{2}\right) - 2 = 10\left(\frac{1}{16}\right) + 41\left(\frac{1}{8}\right) + 12\left(\frac{1}{4}\right) - 7\left(\frac{1}{2}\right) - 2 =$$

$$\frac{5}{8} + \frac{41}{8} + 3 - \frac{7}{2} - 2 =$$

We begin evaluating $p(x)$ at possible roots which have the smallest denominators, so as to keep our fractional arithmetic as simple as possible:

$$p\left(\frac{1}{2}\right) = 10\left(\frac{1}{2}\right)^4 + 41\left(\frac{1}{2}\right)^3 + 12\left(\frac{1}{2}\right)^2 - 7\left(\frac{1}{2}\right) - 2 = 10\left(\frac{1}{16}\right) + 41\left(\frac{1}{8}\right) + 12\left(\frac{1}{4}\right) - 7\left(\frac{1}{2}\right) - 2 =$$

$$\frac{5}{8} + \frac{41}{8} + 3 - \frac{7}{2} - 2 = \frac{5}{8} + (\frac{41}{8}) + \frac{24}{8} - \frac{28}{8} - \frac{16}{8} =$$

We begin evaluating $p(x)$ at possible roots which have the smallest denominators, so as to keep our fractional arithmetic as simple as possible:

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$$\frac{5 + 41 + 24 - 28 - 16}{8} = \frac{26}{8} = \frac{13}{4}$$, so there is a root between 0 and $\frac{1}{2}$.

$$p\left(-\frac{1}{2}\right) =$$

We begin evaluating $p(x)$ at possible roots which have the smallest denominators, so as to keep our fractional arithmetic as simple as possible:

$$p\left(\frac{1}{2}\right) = 10\left(\frac{1}{2}\right)^4 + 41\left(\frac{1}{2}\right)^3 + 12\left(\frac{1}{2}\right)^2 - 7\left(\frac{1}{2}\right) - 2 = 10\left(\frac{1}{16}\right) + 41\left(\frac{1}{8}\right) + 12\left(\frac{1}{4}\right) - 7\left(\frac{1}{2}\right) - 2 =$$

$$\frac{5}{8} + \frac{41}{8} + 3 - \frac{7}{2} - 2 = \frac{5}{8} + (\frac{41}{8}) + \frac{24}{8} - \frac{28}{8} - \frac{16}{8} =$$

$\dfrac{5+41+24-28-16}{8} = \dfrac{26}{8} = \dfrac{13}{4}$, so there is a root between 0 and $\dfrac{1}{2}$.

$$p\left(-\frac{1}{2}\right) = 10\left(-\frac{1}{2}\right)^4 + 41\left(-\frac{1}{2}\right)^3 + 12\left(-\frac{1}{2}\right)^2 - 7\left(-\frac{1}{2}\right) - 2 =$$

We begin evaluating $p(x)$ at possible roots which have the smallest denominators, so as to keep our fractional arithmetic as simple as possible:

$$p\left(\frac{1}{2}\right) = 10\left(\frac{1}{2}\right)^4 + 41\left(\frac{1}{2}\right)^3 + 12\left(\frac{1}{2}\right)^2 - 7\left(\frac{1}{2}\right) - 2 = 10\left(\frac{1}{16}\right) + 41\left(\frac{1}{8}\right) + 12\left(\frac{1}{4}\right) - 7\left(\frac{1}{2}\right) - 2 =$$

$$\frac{5}{8} + \frac{41}{8} + 3 - \frac{7}{2} - 2 = \frac{5}{8} + \left(\frac{41}{8}\right) + \frac{24}{8} - \frac{28}{8} - \frac{16}{8} =$$

$$\frac{5 + 41 + 24 - 28 - 16}{8} = \frac{26}{8} = \frac{13}{4}$$, so there is a root between 0 and $\frac{1}{2}$.

$$p\left(-\frac{1}{2}\right) = 10\left(-\frac{1}{2}\right)^4 + 41\left(-\frac{1}{2}\right)^3 + 12\left(-\frac{1}{2}\right)^2 - 7\left(-\frac{1}{2}\right) - 2 = 10\left(\frac{1}{16}\right) + 41\left(-\frac{1}{8}\right) + 12\left(\frac{1}{4}\right) - 7\left(-\frac{1}{2}\right) - 2 =$$

We begin evaluating $p(x)$ at possible roots which have the smallest denominators, so as to keep our fractional arithmetic as simple as possible:

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$\dfrac{5 + 41 + 24 - 28 - 16}{8} = \dfrac{26}{8} = \dfrac{13}{4}$, so there is a root between 0 and $\dfrac{1}{2}$.

$$p\left(-\frac{1}{2}\right) = 10\left(-\frac{1}{2}\right)^4 + 41\left(-\frac{1}{2}\right)^3 + 12\left(-\frac{1}{2}\right)^2 - 7\left(-\frac{1}{2}\right) - 2 = 10\left(\frac{1}{16}\right) + 41\left(-\frac{1}{8}\right) + 12\left(\frac{1}{4}\right) - 7\left(-\frac{1}{2}\right) - 2 =$$

$$\frac{5}{8} - \left(\frac{41}{8}\right) + \frac{24}{8} + \frac{28}{8} - \frac{16}{8} =$$

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$$p\left(\frac{1}{2}\right) = 10\left(\frac{1}{2}\right)^4 + 41\left(\frac{1}{2}\right)^3 + 12\left(\frac{1}{2}\right)^2 - 7\left(\frac{1}{2}\right) - 2 = 10\left(\frac{1}{16}\right) + 41\left(\frac{1}{8}\right) + 12\left(\frac{1}{4}\right) - 7\left(\frac{1}{2}\right) - 2 =$$

$$\frac{5}{8} + \frac{41}{8} + 3 - \frac{7}{2} - 2 = \frac{5}{8} + (\frac{41}{8}) + \frac{24}{8} - \frac{28}{8} - \frac{16}{8} =$$

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$$p\left(-\frac{1}{2}\right) = 10\left(-\frac{1}{2}\right)^4 + 41\left(-\frac{1}{2}\right)^3 + 12\left(-\frac{1}{2}\right)^2 - 7\left(-\frac{1}{2}\right) - 2 = 10\left(\frac{1}{16}\right) + 41\left(-\frac{1}{8}\right) + 12\left(\frac{1}{4}\right) - 7\left(-\frac{1}{2}\right) - 2 =$$

$$\frac{5}{8} - (\frac{41}{8}) + \frac{24}{8} + \frac{28}{8} - \frac{16}{8} = \frac{5 - 41 + 24 + 28 - 16}{8} = \frac{0}{8} = 0.$$

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$$p\left(-\frac{1}{2}\right) = 10\left(-\frac{1}{2}\right)^4 + 41\left(-\frac{1}{2}\right)^3 + 12\left(-\frac{1}{2}\right)^2 - 7\left(-\frac{1}{2}\right) - 2 = 10\left(\frac{1}{16}\right) + 41\left(-\frac{1}{8}\right) + 12\left(\frac{1}{4}\right) - 7\left(-\frac{1}{2}\right) - 2 =$$

$$\frac{5}{8} - (\frac{41}{8}) + \frac{24}{8} + \frac{28}{8} - \frac{16}{8} = \frac{5 - 41 + 24 + 28 - 16}{8} = \frac{0}{8} = 0.$$

Before looking for any more roots of the polynomial, we may simplify by dividing it by $\left(x - \left(-\frac{1}{2}\right)\right) = x + \frac{1}{2}$, but it is even better to divide by 2 times this factor:

$$
\begin{array}{r|rrrrr}
 & & 5x^3 & +18x^2 & -3x & -2 \\
\hline
2x+1 & 10x^4 & +41x^3 & +12x^2 & -7x & -2 \\
 & 10x^4 & +5x^3 & & & \\
\hline
 & & 36x^3 & +12x^2 & -7x & -2 \\
 & & 36x^3 & +18x^2 & & \\
\hline
 & & & -6x^2 & -7x & -2 \\
 & & & -6x^2 & -3x & \\
\hline
 & & & & -4x & -2 \\
 & & & & -4x & -2 \\
\hline
 & & & & & 0
\end{array}
$$

$$
\begin{array}{r|rrrrr}
 & & 5x^3 & +18x^2 & -3x & -2 \\
\hline
2x+1 & 10x^4 & +41x^3 & +12x^2 & -7x & -2 \\
 & 10x^4 & +5x^3 & & & \\
\cline{2-4}
 & & 36x^3 & +12x^2 & -7x & -2 \\
 & & 36x^3 & +18x^2 & & \\
\cline{3-5}
 & & & -6x^2 & -7x & -2 \\
 & & & -6x^2 & -3x & \\
\cline{3-5}
 & & & & -4x & -2 \\
 & & & & -4x & -2 \\
\cline{4-6}
 & & & & & 0
\end{array}
$$

So now we have $10x^4 + 41x^3 + 12x^2 - 7x - 2 = (2x + 1)(5x^3 + 18x^2 - 3x - 2)$, so we need only search for the possible roots of $q(x) = 5x^3 + 18x^2 - 3x - 2$ that have not already failed for $p(x)$.

$$
\begin{array}{r|rrrrr}
 & & 5x^3 & +18x^2 & -3x & -2 \\ \hline
2x+1 & 10x^4 & +41x^3 & +12x^2 & -7x & -2 \\
 & 10x^4 & +5x^3 & & & \\ \cline{2-4}
 & & 36x^3 & +12x^2 & -7x & -2 \\
 & & 36x^3 & +18x^2 & & \\ \cline{3-5}
 & & & -6x^2 & -7x & -2 \\
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 & & & & -4x & -2 \\
 & & & & -4x & -2 \\ \cline{4-6}
 & & & & & 0
\end{array}
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So now we have $10x^4 + 41x^3 + 12x^2 - 7x - 2 = (2x + 1)(5x^3 + 18x^2 - 3x - 2)$, so we need only search for the possible roots of $q(x) = 5x^3 + 18x^2 - 3x - 2$ that have not already failed for $p(x)$. That leaves us with $\pm\frac{1}{5}$ and $\pm\frac{2}{5}$.

$$p\left(\frac{1}{5}\right) =$$

$$p\left(\frac{1}{5}\right) = 5\left(\frac{1}{5}\right)^3 + 18\left(\frac{1}{5}\right)^2 - 3\left(\frac{1}{5}\right) - 2 =$$

$$p\left(\frac{1}{5}\right) = 5\left(\frac{1}{5}\right)^3 + 18\left(\frac{1}{5}\right)^2 - 3\left(\frac{1}{5}\right) - 2 = 5\left(\frac{1}{125}\right) + 18\left(\frac{1}{25}\right) - 3\left(\frac{1}{5}\right) - 2 =$$

$$p\left(\frac{1}{5}\right) = 5\left(\frac{1}{5}\right)^3 + 18\left(\frac{1}{5}\right)^2 - 3\left(\frac{1}{5}\right) - 2 = 5\left(\frac{1}{125}\right) + 18\left(\frac{1}{25}\right) - 3\left(\frac{1}{5}\right) - 2 =$$

$$\frac{1}{25} + \frac{18}{25} - \frac{3}{5} - 2 =$$

$$p\left(\frac{1}{5}\right) = 5\left(\frac{1}{5}\right)^3 + 18\left(\frac{1}{5}\right)^2 - 3\left(\frac{1}{5}\right) - 2 = 5\left(\frac{1}{125}\right) + 18\left(\frac{1}{25}\right) - 3\left(\frac{1}{5}\right) - 2 =$$

$$\frac{1}{25} + \frac{18}{25} - \frac{3}{5} - 2 = \frac{1}{25} + \frac{18}{25} - \frac{15}{25} - \frac{50}{25} =$$

$$p\left(\frac{1}{5}\right) = 5\left(\frac{1}{5}\right)^3 + 18\left(\frac{1}{5}\right)^2 - 3\left(\frac{1}{5}\right) - 2 = 5\left(\frac{1}{125}\right) + 18\left(\frac{1}{25}\right) - 3\left(\frac{1}{5}\right) - 2 =$$

$$\frac{1}{25} + \frac{18}{25} - \frac{3}{5} - 2 = \frac{1}{25} + \frac{18}{25} - \frac{15}{25} - \frac{50}{25} =$$

$$-\frac{46}{25}$$

$$p\left(\frac{1}{5}\right) = 5\left(\frac{1}{5}\right)^3 + 18\left(\frac{1}{5}\right)^2 - 3\left(\frac{1}{5}\right) - 2 = 5\left(\frac{1}{125}\right) + 18\left(\frac{1}{25}\right) - 3\left(\frac{1}{5}\right) - 2 =$$

$$\frac{1}{25} + \frac{18}{25} - \frac{3}{5} - 2 = \frac{1}{25} + \frac{18}{25} - \frac{15}{25} - \frac{50}{25} =$$

$$-\frac{46}{25}$$

$$p\left(-\frac{1}{5}\right) =$$

$$p\left(\frac{1}{5}\right) = 5\left(\frac{1}{5}\right)^3 + 18\left(\frac{1}{5}\right)^2 - 3\left(\frac{1}{5}\right) - 2 = 5\left(\frac{1}{125}\right) + 18\left(\frac{1}{25}\right) - 3\left(\frac{1}{5}\right) - 2 =$$

$$\frac{1}{25} + \frac{18}{25} - \frac{3}{5} - 2 = \frac{1}{25} + \frac{18}{25} - \frac{15}{25} - \frac{50}{25} =$$

$$-\frac{46}{25}$$

$$p\left(-\frac{1}{5}\right) = 5\left(-\frac{1}{5}\right)^3 + 18\left(-\frac{1}{5}\right)^2 - 3\left(-\frac{1}{5}\right) - 2 =$$

$$p\left(\frac{1}{5}\right) = 5\left(\frac{1}{5}\right)^3 + 18\left(\frac{1}{5}\right)^2 - 3\left(\frac{1}{5}\right) - 2 = 5\left(\frac{1}{125}\right) + 18\left(\frac{1}{25}\right) - 3\left(\frac{1}{5}\right) - 2 =$$

$$\frac{1}{25} + \frac{18}{25} - \frac{3}{5} - 2 = \frac{1}{25} + \frac{18}{25} - \frac{15}{25} - \frac{50}{25} =$$

$$-\frac{46}{25}$$

$$p\left(-\frac{1}{5}\right) = 5\left(-\frac{1}{5}\right)^3 + 18\left(-\frac{1}{5}\right)^2 - 3\left(-\frac{1}{5}\right) - 2 = -5\left(\frac{1}{125}\right) + 18\left(\frac{1}{25}\right) + 3\left(\frac{1}{5}\right) - 2 =$$

$$p\left(\frac{1}{5}\right) = 5\left(\frac{1}{5}\right)^3 + 18\left(\frac{1}{5}\right)^2 - 3\left(\frac{1}{5}\right) - 2 = 5\left(\frac{1}{125}\right) + 18\left(\frac{1}{25}\right) - 3\left(\frac{1}{5}\right) - 2 =$$

$$\frac{1}{25} + \frac{18}{25} - \frac{3}{5} - 2 = \frac{1}{25} + \frac{18}{25} - \frac{15}{25} - \frac{50}{25} =$$

$$-\frac{46}{25}$$

$$p\left(-\frac{1}{5}\right) = 5\left(-\frac{1}{5}\right)^3 + 18\left(-\frac{1}{5}\right)^2 - 3\left(-\frac{1}{5}\right) - 2 = -5\left(\frac{1}{125}\right) + 18\left(\frac{1}{25}\right) + 3\left(\frac{1}{5}\right) - 2 =$$

$$-\frac{1}{25} + \frac{18}{25} + \frac{3}{5} - 2 =$$

$$p\left(\frac{1}{5}\right) = 5\left(\frac{1}{5}\right)^3 + 18\left(\frac{1}{5}\right)^2 - 3\left(\frac{1}{5}\right) - 2 = 5\left(\frac{1}{125}\right) + 18\left(\frac{1}{25}\right) - 3\left(\frac{1}{5}\right) - 2 =$$

$$\frac{1}{25} + \frac{18}{25} - \frac{3}{5} - 2 = \frac{1}{25} + \frac{18}{25} - \frac{15}{25} - \frac{50}{25} =$$

$$-\frac{46}{25}$$

$$p\left(-\frac{1}{5}\right) = 5\left(-\frac{1}{5}\right)^3 + 18\left(-\frac{1}{5}\right)^2 - 3\left(-\frac{1}{5}\right) - 2 = -5\left(\frac{1}{125}\right) + 18\left(\frac{1}{25}\right) + 3\left(\frac{1}{5}\right) - 2 =$$

$$-\frac{1}{25} + \frac{18}{25} + \frac{3}{5} - 2 = -\frac{1}{25} + \frac{18}{25} + \frac{15}{25} - \frac{50}{25} =$$

$$p\left(\frac{1}{5}\right) = 5\left(\frac{1}{5}\right)^3 + 18\left(\frac{1}{5}\right)^2 - 3\left(\frac{1}{5}\right) - 2 = 5\left(\frac{1}{125}\right) + 18\left(\frac{1}{25}\right) - 3\left(\frac{1}{5}\right) - 2 =$$

$$\frac{1}{25} + \frac{18}{25} - \frac{3}{5} - 2 = \frac{1}{25} + \frac{18}{25} - \frac{15}{25} - \frac{50}{25} =$$

$$-\frac{46}{25}$$

$$p\left(-\frac{1}{5}\right) = 5\left(-\frac{1}{5}\right)^3 + 18\left(-\frac{1}{5}\right)^2 - 3\left(-\frac{1}{5}\right) - 2 = -5\left(\frac{1}{125}\right) + 18\left(\frac{1}{25}\right) + 3\left(\frac{1}{5}\right) - 2 =$$

$$-\frac{1}{25} + \frac{18}{25} + \frac{3}{5} - 2 = -\frac{1}{25} + \frac{18}{25} + \frac{15}{25} - \frac{50}{25} =$$

$$-\frac{18}{25}$$

$$p\left(\frac{1}{5}\right) = 5\left(\frac{1}{5}\right)^3 + 18\left(\frac{1}{5}\right)^2 - 3\left(\frac{1}{5}\right) - 2 = 5\left(\frac{1}{125}\right) + 18\left(\frac{1}{25}\right) - 3\left(\frac{1}{5}\right) - 2 =$$

$$\frac{1}{25} + \frac{18}{25} - \frac{3}{5} - 2 = \frac{1}{25} + \frac{18}{25} - \frac{15}{25} - \frac{50}{25} =$$

$$-\frac{46}{25}$$

$$p\left(-\frac{1}{5}\right) = 5\left(-\frac{1}{5}\right)^3 + 18\left(-\frac{1}{5}\right)^2 - 3\left(-\frac{1}{5}\right) - 2 = -5\left(\frac{1}{125}\right) + 18\left(\frac{1}{25}\right) + 3\left(\frac{1}{5}\right) - 2 =$$

$$-\frac{1}{25} + \frac{18}{25} + \frac{3}{5} - 2 = -\frac{1}{25} + \frac{18}{25} + \frac{15}{25} - \frac{50}{25} =$$

$$-\frac{18}{25}$$

$$p\left(\frac{2}{5}\right) =$$

$$p\left(\frac{1}{5}\right) = 5\left(\frac{1}{5}\right)^3 + 18\left(\frac{1}{5}\right)^2 - 3\left(\frac{1}{5}\right) - 2 = 5\left(\frac{1}{125}\right) + 18\left(\frac{1}{25}\right) - 3\left(\frac{1}{5}\right) - 2 =$$

$$\frac{1}{25} + \frac{18}{25} - \frac{3}{5} - 2 = \frac{1}{25} + \frac{18}{25} - \frac{15}{25} - \frac{50}{25} =$$

$$-\frac{46}{25}$$

$$p\left(-\frac{1}{5}\right) = 5\left(-\frac{1}{5}\right)^3 + 18\left(-\frac{1}{5}\right)^2 - 3\left(-\frac{1}{5}\right) - 2 = -5\left(\frac{1}{125}\right) + 18\left(\frac{1}{25}\right) + 3\left(\frac{1}{5}\right) - 2 =$$

$$-\frac{1}{25} + \frac{18}{25} + \frac{3}{5} - 2 = -\frac{1}{25} + \frac{18}{25} + \frac{15}{25} - \frac{50}{25} =$$

$$-\frac{18}{25}$$

$$p\left(\frac{2}{5}\right) = 5\left(\frac{2}{5}\right)^3 + 18\left(\frac{2}{5}\right)^2 - 3\left(\frac{2}{5}\right) - 2 =$$

$$p\left(\frac{1}{5}\right) = 5\left(\frac{1}{5}\right)^3 + 18\left(\frac{1}{5}\right)^2 - 3\left(\frac{1}{5}\right) - 2 = 5\left(\frac{1}{125}\right) + 18\left(\frac{1}{25}\right) - 3\left(\frac{1}{5}\right) - 2 =$$

$$\frac{1}{25} + \frac{18}{25} - \frac{3}{5} - 2 = \frac{1}{25} + \frac{18}{25} - \frac{15}{25} - \frac{50}{25} =$$

$$-\frac{46}{25}$$

$$p\left(-\frac{1}{5}\right) = 5\left(-\frac{1}{5}\right)^3 + 18\left(-\frac{1}{5}\right)^2 - 3\left(-\frac{1}{5}\right) - 2 = -5\left(\frac{1}{125}\right) + 18\left(\frac{1}{25}\right) + 3\left(\frac{1}{5}\right) - 2 =$$

$$-\frac{1}{25} + \frac{18}{25} + \frac{3}{5} - 2 = -\frac{1}{25} + \frac{18}{25} + \frac{15}{25} - \frac{50}{25} =$$

$$-\frac{18}{25}$$

$$p\left(\frac{2}{5}\right) = 5\left(\frac{2}{5}\right)^3 + 18\left(\frac{2}{5}\right)^2 - 3\left(\frac{2}{5}\right) - 2 =$$

$$5\left(\frac{8}{125}\right) + 18\left(\frac{4}{25}\right) - 3\left(\frac{2}{5}\right) - 2 =$$

$$p\left(\frac{1}{5}\right) = 5\left(\frac{1}{5}\right)^3 + 18\left(\frac{1}{5}\right)^2 - 3\left(\frac{1}{5}\right) - 2 = 5\left(\frac{1}{125}\right) + 18\left(\frac{1}{25}\right) - 3\left(\frac{1}{5}\right) - 2 =$$

$$\frac{1}{25} + \frac{18}{25} - \frac{3}{5} - 2 = \frac{1}{25} + \frac{18}{25} - \frac{15}{25} - \frac{50}{25} =$$

$$-\frac{46}{25}$$

$$p\left(-\frac{1}{5}\right) = 5\left(-\frac{1}{5}\right)^3 + 18\left(-\frac{1}{5}\right)^2 - 3\left(-\frac{1}{5}\right) - 2 = -5\left(\frac{1}{125}\right) + 18\left(\frac{1}{25}\right) + 3\left(\frac{1}{5}\right) - 2 =$$

$$-\frac{1}{25} + \frac{18}{25} + \frac{3}{5} - 2 = -\frac{1}{25} + \frac{18}{25} + \frac{15}{25} - \frac{50}{25} =$$

$$-\frac{18}{25}$$

$$p\left(\frac{2}{5}\right) = 5\left(\frac{2}{5}\right)^3 + 18\left(\frac{2}{5}\right)^2 - 3\left(\frac{2}{5}\right) - 2 =$$

$$5\left(\frac{8}{125}\right) + 18\left(\frac{4}{25}\right) - 3\left(\frac{2}{5}\right) - 2 =$$

$$\frac{8}{25} + \frac{72}{25} - \frac{6}{5} - 2 = 0$$

so now we know that $q(x)$ is divisible by $x - \frac{2}{5}$, so we divide $q(x)$ by $5x - 2$:

$$
\begin{array}{r|rrrr}
 & & x^2 & +4x & +1 \\
\hline
5x-2 & 5x^3 & +18x^2 & -3x & -2 \\
 & 5x^3 & -2x^2 & & \\
\hline
 & & 20x^2 & -3x & -2 \\
 & & 20x^2 & -8x & \\
\hline
 & & & 5x & -2 \\
 & & & 5x & -2 \\
\hline
 & & & & 0
\end{array}
$$

$$
\begin{array}{r|rrrr}
 & & x^2 & +4x & +1 \\
\cline{2-5}
5x-2 & 5x^3 & +18x^2 & -3x & -2 \\
 & 5x^3 & -2x^2 & & \\
\cline{2-4}
 & & 20x^2 & -3x & -2 \\
 & & 20x^2 & -8x & \\
\cline{3-5}
 & & & 5x & -2 \\
 & & & 5x & -2 \\
\cline{3-5}
 & & & & 0
\end{array}
$$

We observe that the roots of x^2+4x+1 are $\frac{-4 \pm \sqrt{4^2-4(1)(1)}}{2(1)} = \frac{-4 \pm \sqrt{12}}{2} = \frac{-4 \pm 2\sqrt{3}}{2} = -2 \pm \sqrt{3}$, so the factorization of $p(x)$ is

$$
\begin{array}{r|rrrr}
 & x^2 & +4x & +1 & \\
\hline
5x-2 & 5x^3 & +18x^2 & -3x & -2 \\
 & 5x^3 & -2x^2 & & \\
\hline
 & & 20x^2 & -3x & -2 \\
 & & 20x^2 & -8x & \\
\hline
 & & & 5x & -2 \\
 & & & 5x & -2 \\
\hline
 & & & & 0
\end{array}
$$

We observe that the roots of $x^2 + 4x + 1$ are $\dfrac{-4 \pm \sqrt{4^2 - 4(1)(1)}}{2(1)} = \dfrac{-4 \pm \sqrt{12}}{2} = \dfrac{-4 \pm 2\sqrt{3}}{2} = -2 \pm \sqrt{3}$, so the factorization of $p(x)$ is

$p(x) = (2x+1)(5x-2)(x^2+4x+1) =$

$$
\begin{array}{r|rrrr}
 & x^2 & +4x & +1 & \\
\hline
5x-2 & 5x^3 & +18x^2 & -3x & -2 \\
 & 5x^3 & -2x^2 & & \\
\hline
 & & 20x^2 & -3x & -2 \\
 & & 20x^2 & -8x & \\
\hline
 & & & 5x & -2 \\
 & & & 5x & -2 \\
\hline
 & & & & 0
\end{array}
$$

We observe that the roots of $x^2 + 4x + 1$ are $\dfrac{-4 \pm \sqrt{4^2 - 4(1)(1)}}{2(1)} = \dfrac{-4 \pm \sqrt{12}}{2} = \dfrac{-4 \pm 2\sqrt{3}}{2} = -2 \pm \sqrt{3}$, so the factorization of $p(x)$ is

$p(x) = (2x+1)(5x-2)(x^2+4x+1) =$

$(2x+1)(5x-2)\left(x-(-2-\sqrt{3})\right)\left(x-(-2+\sqrt{3})\right)$

When we have learned a little about Calculus, we will be able to shorten this process considerably.