Report on Sabbatical Leave
(July 1, 2003 to June 30, 2004)

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The first seven weeks of my sabbatical were spent at the U of S in Saskatoon. From August 16 to September 7, 2003, I was at the Banff International Research Station for a three weeks **Research in Teams**. From September 8, 2003 to June 30th, 2004, I was based at the **Institut de Mathématiques de Jussieu** (Université Paris 6/7) as a member of the "Equipe Géométrie et Dynamique", and at the **Laboratoire de Mathématiques de l'Université de Versailles Saint-Quentin-en-Yvelines** as a member of the "Equipe Algèbre-Géométrie". I also frequently worked with members of the **Equipe de Logique Mathématique** at Université Paris 7, which is affiliated with the "Centre National de la Recherche Scientifique" of France, and gave talks in their seminars.

During my sabbatical I **organized a conference** at the Institut Henri Poincaré in Paris, **attended two conferences** related to my research (in Tehran and Pisa) and **visited** the universities of Shiraz, Cairo, Freiburg, Konstanz, Angers, Darmstadt and Zagreb, to report on my research, collaborate with co-authors, or initiate new collaborations.

In addition, I worked on several papers, and I supervised a DEA student at the University of Versailles. Below, I will discuss in detail the work that I undertook at every stage of this sabbatical.

1 Saskatoon, July and August 2003

I supervised an NSERC Summer Award student:

Kelly Skoye, U of S, for a project on *Finite fields and coding theory*. I met with the student at least once a week while I was still in Saskatoon. Afterwards, Kelly was supervised by Murray Marshall, and I was in email contact with her. Kelly did a research of the existing literature and compiled a detailed and commented bibliography on coding theory (6 pages, available upon request), which will help me in developing a course on coding theory. Here is an excerpt from the NSERC application:

> Because of its important applications in computer science, coding theory is a hot topic in mathematical research. The project shall introduce the student to this area of research and encourage her to undertake graduate studies and pursue a research career in pure mathematics. Through the literature research and the bibliography, the project will also be beneficial for the supervisor, who is presently getting interested in research in cryptography and coding theory, because of its connections to algebraic function fields.

The remaining time at Saskatoon was devoted to the preparation of my stay in Paris, and of the upcoming Research in Teams at BIRS.

2 Research in Teams at BIRS

Together with Dale Cutkosky (U. of Missouri at Columbia), I organized a three weeks **Research in Teams** on *Local uniformization and resolution of singularities* at the new Banff International Research Station. Further participants were Shreeram Abhyankar (Purdue University), Hagen Knaf (Fraunhofer Institut für Techno- und Wirtschaftsmathematik, Kaiserslautern), and Bernard Teissier (Institut de Mathématiques de Jussieu). See the web site

`http://www.pims.math.ca/birs/workshops/2003/03rit006`

Right now, we are at a point where we see much more clearly than before the main ramification theoretic problems which play a decisive role in the following problems:
(1) resolution of singularities in positive characteristic, and
(2) the model theory of valued fields in positive characteristic.

Notions like the “defect” have been defined and studied extensively, and the structure of algebraic function fields with valuations has been understood in greater depth. At this point, it is highly beneficial to bring together and unite the experience of some of the ”key players” in this development. Although their methods are quite similar, we still have to put effort in comparing and combining them.

Since the Valuation Theory Conference, Saskatoon 1999, and the Conference in honour of Abhyankar’s 70th birthday, Purdue 2000, there is a revived confidence that valuation theory could help to solve the problem of resolution of singularities in positive characteristic. Therefore, we think that it is important now to combine the invaluable experience of Abhyankar with the new valuation theoretical ideas of Cutkosky, Teissier, myself and others. The goal is: (a) to push our techniques further and derive new results, (b) to formulate key questions, and consequently, a research program for the solution of problems (1) and (2).

The final report of our Research in Teams is enclosed as an attachment to this report.

With Hagen Knaf, I worked on our second joint paper *Every place admits local uniformization in a finite extension of the function field*. We convinced ourselves that like in our first joint paper (mentioned under 3.4 below), the arithmetic case can successfully be treated, in addition to the local uniformization I had previously worked out in a single-authored paper of the same name (which had not yet been published). This is to say that local uniformization in a finite extension of the function field can be proved by valuation theoretical means also over certain ground rings, not only over fields. We worked further on this paper whenever we met in the past academic year. This paper should be finished before the end of 2004.

On September 6 and 7, Hagen Knaf and I met with the incoming Focused Research Group on *Arithmetic of fundamental groups* which was organized by David Harbater (U. of Pennsylvania) and Florian Pop (U. of Bonn). (In fact, I was responsible for the existence of this Focused Research Group as I had urged the organizers to apply at BIRS.) On September 7, I gave a survey talk in this group on the application of ramification theory and the theory of valued function fields to the problem of local uniformization.

3 Paris and France, September 2003 to June 2004

3.1 Equipe Géométrie et Dynamique, Paris

Several of our discussions during the Research in Teams at BIRS were continued during my stay in Paris. For example, at the Research in Teams we had identified the problem of an Implicit Function Theorem in infinitely many variables which could be used in Bernard Teissier's "characteristic blind" approach to local uniformization. We had convinced ourselves that the theory of mappings on ultrametric spaces worked out in my paper *Maps on ultrametric spaces, Hensel's Lemma, and differential equations over valued fields* can be applied to prove such a theorem. After further discussions with Teissier in Paris, I came up with a theorem that appears to do the job (the details of its application have yet to be worked out by Teissier). The above mentioned paper of 37 pages had been accepted for publication in Communications in Algebra, but the referee had made many nice suggestions for its improvement, so I was already working on a major revision of the paper. I then decided to adapt the paper to cover also the new theorem. The completion of this revision is one of the urgent projects which will be continued right after the submission of this sabbatical report.

With Teissier and Olivier Piltant, I discussed details of Teissier's approach to local uniformization, which is work in progress. Teissier had published a very long paper in our Conference Proceedings of the 1999 Valuation Theory Conference (see 6 below), where several proofs remain to be worked out. I provided some examples which could be used to check critical cases, and a counterexample to a conjecture of Teissier. Frotunately, this conjecture is not essential for his approach.

In early 2004, I started to work on a paper called *Classification of Artin Schreier defect extensions, and characterizations of algebraically maximal and defectless fields.* What had been intended to be "just" publishing a nice and relatively isolated result from my own thesis (which is attracting some attention from other researchers right now), has become a major work which has devoured many weeks of exclusive work. The present version of the paper has 50 pages. This developments has two reasons.

First, the classification results of my thesis all referred to henselian fields. But the classification is of particular interest for the case of function fields, as recent work of Cutkosky and Piltant has shown (and function fields with non-trivial valuations are never henselian). So I decided to generalize everything to the case on non-henselian fields. I intended to submit the paper while I was still in Paris, but in the last moment I found a gap in the generalization. This gap can be closed, but that requires some additional effort.

Second, the discussion of maximality properties of valued fields is connected in a natural way with the original subject of the paper. So I decided to do this discussion systematically, thereby filling several gaps in proofs that are found in the literature.

The discussions with Teissier helped me to identify further interesting aspects of the subject, such as the deformation of Artin-Schreier extensions. So I included these aspects as well. Here is the present abstract of the paper:

> We classify Artin-Schreier extensions of valued fields with non-trivial defect according to whether they are connected with purely inseparable extensions with non-trivial defect, or not. We use this classification to show that in positive characteristic, a valued field is algebraically complete if and only if it has no proper immediate algebraic extension and every finite purely inseparable extension is defectless. This result is an important tool for the construction of algebraically complete fields. We also use the result to show that extremal fields are algebraically complete. A valued field (K, v) is called extremal if for all polynomials f in several variables the value set $vf(K^n)$ has a maximum. Restricting this condition to certain classes of polynomials yields further interesting properties. In that way, we give characterizations of algebraically maximal and inseparably defectless fields. Finally, we give a second characterization of algebraically complete fields, in terms of their completion.
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> As an example by Cutkosky and Piltant shows, a certain property called relative resolution may work with one type of Artin Schreier defect extensions, but not with the other. This connection with algebraic geometry has to be investigated further.
>
> Initially, this work was strongly inspired by the first part of Francoise Delon's thesis. Some results are generalized, some others are put in a larger perspective.

Together with Bernard Teissier and Charles Favre, I organized the Séminaire sur les Singularités of the Equipe. Its program can be found at:

`http://www.math.jussieu.fr/projets/gdy/singularites/programme.html`

In this seminar, I gave a series of four talks on *Valuation theory in positive characteristic and the problem of local uniformization.* Here is the abstract for these talks:

> I will describe the valuation theoretic formulation of the problem of local uniformization. Then I will give an overview of its relation to the following

subjects (which will be discussed in more detail in later talks):
- the defect of valued field extensions
- the structure of valued function fields: the Generalized Stability Theorem and henselian rationality
- the model theory of valued fields
- additive polynomials, extremal fields
- valuations on rational function fields.

3.2 Equipe Algèbre-Géométrie, Versailles

I visited the University of Versailles in average once a week. Together with Vincent Cossart, Monique Lejeune-Jalabert and Olivier Piltant, I worked on several subjects, the most important of which is Cossart's recent work on resolution of singularities in dimension 3 in positive characteristic, in particular the case of Artin-Schreier extensions. I regularly attended the seminar of the Equipe Algèbre-Géométrie and contributed two talks: *Resolution of singularities and the model theory of valued fields* and *The Zariski space of an algebraic function field, rational places and large fields*. For the program of our working group and the seminar, see the web pages

`http://fermat.math.uvsq.fr/lama/algebre/actrec_a/groupag0304.html`
`http://fermat.math.uvsq.fr/lama/algebre/actrec_a/semag0304.html`

I also gave a series of six talks *A quick introduction to model theoretic algebra*. Here is the abstract for this mini-course:

> I will give an introduction to the methods of model theoretic algebra, with several examples of applications (mainly to fields and ordered abelian groups). I will define the basic notions, like "language", "formula", "sentence", "elementary", "theory", "structure", "model", "elementary equivalent", "existentially closed", "complete", "model complete", "decidable", "quantifier elimination", "saturated". I will discuss the compactness theorem, existence of saturated models, the use of embedding lemmas, and Robinson's Test for model completeness. As main applications, I will present the framework of "Nullstellensaetze", transfer principles like the "Poor Man's Lefschetz Principle", and the "Ax-Kochen-Ershov Principle" for valued fields. If time permits, I will give a quick overview of the known model theoretic results and open problems for valued fields.

I started a joint project with Piltant, who is now mainly based at Valladolid in Spain, but visited Versailles frequently. The project is based on my above mentioned paper *Classification of Artin Schreier defect extensions...* and continues the work by determining the higher ramification groups of such extensions. We are also working on a generalization to extensions generated by other additive polynomials. The idea is that the classification given in my paper can also be read off from the higher ramification groups, which are of interest in algebraic geometry and thus constitute an instance where the classification could be applied.

During my sabbatical stay I have supervised Marc Autord, a DEA student at the University of Versailles. I was member of the jury of his oral examination, July 2004. I have refereed and corrected his DEA thesis *Théorème d'Ax–Kochen: Valuations et modèles*, which he submitted in Summer of 2004. On the basis of the oral examination and his thesis, Marc Autord was awarded the DEA in the Fall of 2004.

3.3 Equipe de Logique Mathématique

In the general seminar of the Equipe, I gave a talk on the *Model Theory of valued fields in positive characteristic: results and open questions*. Here is the abstract:

> I will describe the main known results about the Model Theory of valued fields in positive characteristic (algebraically closed valued fields, Kaplansky fields, tame fields), and then list the most important open problems. I will talk about a few positive and negative results for power series fields over finite fields. I will discuss the role of additive polynomials and finally introduce the notion of "extremal field" and describe the open questions connected with it.

For the program of the general seminar, see:

`http://www.logique.jussieu.fr/sgen03-04.html`.

In the seminar on ordered algebraic structures, organized by Francoise Delon, Max Dickmann and Danielle Gondard, I gave the following two talks: *Classification of Artin Schreier defect extensions, and characterizations of algebraically maximal and defectless fields* (for the contents, see my discussion of the paper *Classification of Artin Schreier defect extensions...* under 3.1 above), and *Extensions of valuations to rational function fields*. Here is the abstract of the latter talk:

> We classify all possible extensions of a valuation from a ground field K to a rational function field in one or several variables over K. We determine which value groups and residue fields can appear, and we show

how to construct extensions having these value groups and residue fields. In particular, we give constructions of extensions whose corresponding value group and residue field extensions are not finitely generated. One can even construct valuations on rational function fields in two variables which admit an infinite tower of degree p extensions with defect p. Such nasty valuations constitute a serious empediment for local uniformization in positive characteristic, so it is important to study their structure in detail. In the case of a rational function field $K(x)$ in one variable, we consider the relative algebraic closure of K in the henselization of $K(x)$ with respect to the given extension, and we show that this can be any countably generated separable-algebraic extension of K. In the "tame case", we show how to determine this relative algebraic closure. These methods can be applied to power series fields and to the p-adics.

After the conference in Tehran (see 4.1 below) and in connection with my work on dense subfields of real closed fields, I got interested in the existence problem for integer parts. J.-P. Ressayre and H. Mourgues, both members of the Equipe de Logique, had written a paper where they proved the existence of integer parts in real closed fields using truncation-closed embeddings in power series fields. Since this is quite heavy machinery, I thought about a direct proof for the existence, which could also tell us more about the intrinsic properties of integer parts. I frequently attended the seminar organized by Ressayre and Salma Kuhlmann; see

`http://www.logique.jussieu.fr/www.rambaud/GTL.html`.

Working on this problem, I started a paper with the working title *Existence of complements for a valuation ring*, which is work in progress. I reported on my results in two talks *Towards a Direct Proof of the Existence of Integer Parts* in the seminar.

On the occasion of several visits of Gérard Leloup (U. of LeMans) to the Equipe de Logique, we discussed the corrections for our paper *Formal power series with cyclically ordered exponents* (by M. Giraudet, F.-V. Kuhlmann, G. Leloup), which had been accepted in May of 2004 for publication in Archiv der Mathematik. We also started to work on the open problems mentioned in this paper, which appear to be quite difficult. Here is the abstract of our paper:

We define and study a notion of ring $k[[C]]$ of formal power series with exponents in a cyclically ordered group. These power series have well-ordered supports in any linear ordering obtained from the cyclic ordering by cutting at an arbitrary point. Using a theorem of Rieger, we show that such a ring is a quotient of various corresponding subrings of classical formal power series rings. If $l(C)$ denotes the largest totally ordered subgroup of C, then we show that $k[[C]]$ is a field if k is a field, C is torsion free and $C/l(C)$ is finite. Conversely, we prove that if $k[[C]]$ is a field, then k is a field, C is torsion free and $C/l(C)$ embeds in the group of all roots of unity. We show that the polynomial ring $k[C]$ is an integral domain if and only if k is an integral domain and C is torsion free. Further, we characterize all subrings with 1 of $k[C]]$. The ring $k[[C]]$ carries a two variable valuation function. In the particular case where the cyclically ordered group is actually totally ordered, our notion of formal power series is equivalent to the classical one in a language enriched with a predicate interpreted by the set of all monomials.

The dvi, ps and pdf files for this paper can be downloaded from the Valuation Theory Home Page:

`http://math.usask.ca/fvk/recpap.htm` (publication number 127).

3.4 The Fifth Annual Colloquiumfest, IHP, Paris

Together with Zoe Chatzidakis (Université Paris 7) I organized the Fifth Annual Colloquiumfest as a joint venture of the Equipe Géométrie et Dynamique and the Equipe de Logique Mathématique. This conference was a continuation of the four previous Colloquiumfest conferences at Saskatoon, 2000-2003. It took place on April 5 and 6, 2004, at the Institut Henri Poincaré (IHP) in Paris. It was supported by the two equipes and the IHP. This year's emphasis was on *Valuation Theory in Algebraic Geometry and Model Theory*. For the program, see

`http://math.usask.ca/fvk/Mb5.htm`.

The poster for the Fifth Annual Colloquiumfest is enclosed as an attachment to this report.

On the occasion of Hagen Knaf's visit to Paris for the Colloquiumfest, we discussed the corrections suggested by the referee for our paper *Abhyankar places admit local uniformization in any characteristic*, which had been conditionally accepted for publication in Annales Scientifiques de l'Ecole Normale Supérieure earlier in 2004. The dvi, ps and pdf files for this paper can be downloaded from the Valuation Theory

Home Page:

`http://math.usask.ca/fvk/recpap.htm` (publication number 136).

The condition for the publication of the paper is that a second paper be submitted, containing the proof of a main theorem proved in my thesis, which we used in our paper. Therefore, I immediately started the preparation of this new paper, under the working title *Elimination of Wild Ramification I: the Generalized Stability Theorem*. It should be finished within the fall of 2004.

4 Conferences attended

4.1 IPM Tehran, Iran

The first half of my visit to Iran (October 17 to 27, 2003) was devoted to the *Workshop and Conference on Logic, Algebra and Arithmetic* at the Institute for Theoretical Physics and Mathematics Tehran, where I was an invited speaker. For program and schedule of the meeting, see:

`http://www.ipm.ac.ir/logic2003`.

I gave a general introductory talk on *Resolution of singularities and the model theory of valued fields*. Here is the abstract:

> I will give a quick introduction to the basic problem of resolution of singularities and show why its local form (local uniformization) is of valuation theoretical nature. I will give some basic examples of valued fields and explain Hensel's Lemma and its meaning for the model theory of valued fields. I will then explain the connection between Hensel's Lemma, ramification and local uniformization. From there, I will describe some connections between resolution of singularities and the model theory of valued fields, and some open questions about these connections.

The second, more specialized conference talk was on *Additive polynomials and their role in the model theory of power series fields over finite fields and in local uniformization*. Here is the abstract:

> A polynomial f over an infinite field K is called additive if $f(a+b) = f(a)+f(b)$ for all $a, b \in K$. If the characteristic of K is 0, then the only additive polynomials are of the form cx with $c \in K$. But if the characteristic

is $p > 0$, then for instance, X^p and the Artin-Schreier-polynomial $X^p - X$ are additive. I will explain the particular role that additive polynomials play in the model theory of power series fields over finite fields. This is tightly connected with the structure theory of valued function fields, which in turn also plays a crucial role for the problem of local uniformization. The latter is a local form of resolution of singularities and therefore of valuation theoretical nature. While local uniformization has been proved in characteristic 0 by Zariski in 1940, the positive characteristic case is still open, and only special cases have been solved. In all of these solutions, additive polynomials play a role. Finally, I will sketch some results on the classification of certain Artin-Schreier-extensions of valued fields and their connection with recent work of Cutkosky and Piltant on resolution of singularities in positive characteristic.

At the conference, one of the speakers (Mojtaba Moniri) who works on models of arithmetic, where integer parts of real closed fields play an important role, asked the following question: *Does every real closed field admit a proper dense subfield?* This is interesting because if K is a proper dense subfield of L and admits an integer part, then this is also an integer part of L, but its quotient field lies in K and hence is smaller than L. I solved this problem and submitted a paper called *Dense subfields of henselian fields, and integer parts* (17 pages) to the Conference Proceedings. Here is the abstract of this paper:

> We show that every henselian valued field L of residue characteristic 0 admits a proper subfield K which is dense in L. We present conditions under which this can be taken such that $L|K$ is transcendental and K is henselian. These results are of interest for the investigation of integer parts of ordered fields. We present examples of real closed fields which are larger than the quotient fields of all their integer parts. Finally, we give rather simple examples of ordered fields that do not admit any integer part and of valued fields that do not admit any subring which is an additive complement of the valuation ring.

The dvi, ps and pdf files for this paper can be downloaded from the Valuation Theory Home Page:

`http://math.usask.ca/fvk/recpap.htm` (publication number 138).

I reported on the results of this paper at the Pisa conference (see 4.2 below). The preparation of my Pisa conference talk led to more results which I will add to the paper; I am working on the revision right now. I also noticed that one of the approaches used in that paper leads to an amazing counterexample connected with the classical problem of embedding valued fields in power series fields. It had been known that not every equicharacteristic valued field can be embedded in a power series field over their residue field and their value group. But the known counterexamples I found in the literature are fields that are not finitely generated over their residue field. With my new approach I can construct valuations on rational function fields in three variables which conctitute a counterexample. This shows that the complexity of nasty valuations (and nasty orderings derived from them) is quite independent of the complexity of the underlying field. A short paper with working title *Embedding valued rational functions fields in power series fields* is in preparation.

I submitted a second paper to the Proceedings, closely related to my second conference talk: *Additive Polynomials and Their Role in the Model Theory of Valued Fields.* This paper is to some extent a continuation of my introductive and programmatic paper *Valuation theoretic and model theoretic aspects of local uniformization*, which was published in: *Resolution of Singularities — A Research Textbook in Tribute to Oscar Zariski*, Herwig Hauser, Joseph Lipman, Frans Oort, Adolfo Quiros (eds.), Progress in Mathematics Vol. **181**, Birkhäuser Verlag Basel (2000), 381-456. In that paper I had pointed out that the ramification theoretical **defect** of finite extensions of valued fields is responsible for the problems we have when we deal with the model theory of valued fields or try to prove local uniformization in positive characteristic. In the present paper I discuss the connection between the defect and additive polynomials. I state and prove basic facts about additive polynomials and then treat several instances where they enter the theory of valued fields in an essential way that is particularly interesting for model theorists and algebraic geometers. In particular, non-commutative structures (skew polynomial rings) seem to play an essential role in the structure theory of valued fields in positive characteristic. Further, I state the main open questions. For the convenience of the reader, I also include some exercises. The dvi, ps and pdf files for this paper can be downloaded from the Valuation Theory Home Page:

`http://math.usask.ca/fvk/recpap.htm` (publication number 137).

It was also very inspiring to meet Yuri Ershov at the conference. One of his conference talks built on earlier work of mine, and it introduced me to the notion of *extremal field*, which I used in my above mentioned paper on the classification of Artin-Schreier defect extensions. In that paper I am also filling a gap that he had found in a proof in the literature.

4.2 Pisa, Italy

I participated in the International Congress on Nonstandard Models of Arithmetic and Analysis at the University of Pisa (June 24 to 28, 2004); for the program, see

http://www.dm.unipi.it/ dinasso/marian2004.

I gave a contributed talk *Dense subfields and integer parts*. Here is the abstract:

> Answering a question asked at the Conference "Logic, Algebra and Arithmetic" in Teheran 2003, We show that every real closed (or more generally, henselian) non-archimedean ordered field L admits a proper dense subfield. This is interesting because any integer part of K will also be an integer part of L. If L is "small", then $L|K$ must be algebraic. However, there are examples of $\mathrm{trdeg}\, L|K$ being any pre-assigned cardinal. In particular, if the natural valuation on L has no coarsest non-trivial coarsening, then $\mathrm{trdeg}\, L|K$ can be any countable cardinal. At the same time our construction provides a counterexample to the following conjecture: *If a valuation v has no coarsest non-trivial coarsening and if the residue field with respect to every coarsening is henselian, then v is henselian.*
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> We will also discuss the existence of integer parts and of truncation closed embeddings in power series fields. We show that Boughattas' ordered field which admits no integer part is not henselian. Finally, we show that there is a real closed field that has larger cardinality than the quotient fields of all of its integer parts. It is an open question whether there is a real closed field that is larger than the quotient fields of all of its integer parts, but has the same cardinality.

At the conference I met Antongiulio Fornasiero, a post-doctoral fellow of Alessandro Berarducci. At a previous visit to Pisa, Salma Kuhlmann had already met him and had suggested to him to work on a generalization of the above mentioned result by Ressayre and Mourgues on the existence of integer parts (see 3.3). Salma and I conjectured that it can be generalized to henselian ordered fields. Since then, Fornasiero had been in email contact with us, and I had helped him with several valuation theoretical aspects of this problem. He is writing a paper, the preliminary version of which is available at:

http://www.dm.unipi.it/~fornasiero/ressayre.pdf.

We have invited Fornasiero to visit the U of S as a three months short term postdoctoral fellow in the present academic year.

5 Visits to other universities

5.1 University of Shiraz, Iran

After the conference in Tehran (see 4.1 above), I visited the University of Shiraz, where a former student of the U of S algebra group, Mehdi Zekavat, is working as a professor in the Mathematics Department. I gave a colloquium talk on *Some applications of valuation theory*, which was attended by many students. I met with the president of the university who showed a great interest in student exchange with the U of S. I also met with several members of the Mathematics Department and found out that the work of one of them, Habib Sharif, on *E-algebraic functions* has unexpected relations to my own work on additive polynomials. This could be a basis for future collaboration. Later, I discussed this work with my colleagues at Versailles who were also very interested.

The University of Shiraz is planning to organize an algebra conference in 2005 or 2006. There is the possibility that Salma Kuhlmann and myself could become co-organizers.

5.2 Cairo University, Egypt

In late December of 2003, I visited the Mathematics Department of Cairo University and gave a talk on local uniformization and the model theory of valued fields in their Colloquium. At this department there are several colleagues who are particularly interested in the non-commutative aspects of my work on additive polynomials. We agreed to jointly organize a conference on algebra at Cairo University within the next couple of years.

5.3 Universities of Freiburg and Konstanz, Germany

From February 1 to 7, 2004, I visited first the University of Freiburg and then the University of Konstanz. At Freiburg, I gave a talk on *Lokale Uniformisierung von Abhyankarstellen* in the Oberseminar Modelltheorie und Algebra (organized by Martin Ziegler, Freiburg, Alexander Prestel, Konstanz, and Jochen Koenigsmann, Basel). This talk was meant to introduce graduate students to valuation theoretical methods in local uniformization. The continuation of this talk was given in Alexander Prestel's seminar at Konstanz under the title of *Lokale Uniformisierung in positiver Charakteristik nach endlicher Erweiterung des Funktionenkoerpers*. During my stay, I discussed my work with Prestel and his graduate students. I also discussed with Prestel a planned joint application (together with Teissier) for a conference on valuation theory at Oberwolfach.

5.4 University of Angers, France

From May 1 to May 5, 2004, I visited the University of Angers. I gave a Colloquium talk on *Elimination of ramification, local uniformization and the model theory of valued fields*, and discussed details of my work with Adam Parusinski (who is working in singularity theory) and Francois Lucas (who is a collaborator of Mark Spivakovsky).

5.5 University of Darmstadt, Germany

On June 11th, 2004, I visited the University of Darmstadt with which we have a successful student exchange agreement. I met with 13 undergraduate students who are interested to come to Saskatoon, three of whom are now here for the academic year 2004-2005. I informed the students of the details of the exchange program and the academic offerings of the U of S, and gave them practical hints for the life in Saskatoon.

5.6 University of Zagreb, Croatia

Following an invitation by Salma Kuhlmann's collaborator, D. Biljakovic, I gave a talk on July 1, 2004, in the Mathematics Colloquium of the University of Zagreb on *Elimination of ramification, local uniformization and the model theory of valued fields.* I also continued my discussions with Biljakovic and Salma on valuation theoretical aspects of integer parts of ordered fields.

6 Other work done during my sabbatical

In the past academic year, I have **refereed** two papers, including their revisions. Further, I have written two reviews for the **Mathematical Reviews**. I had to check the page proofs of the papers *On places of algebraic function fields in arbitrary characteristic*, which in the fall of 2003 had been accepted for publication in Advances in Mathematics (26 pages), and of the paper *Value groups, residue fields and bad places of rational function fields*, which in 2002 had been accepted for publication in Transactions of the American Mathematical Society (43 pages).

In September of 2003, the second volume of our 1999 Conference Proceedings was published: *Valuation Theory and its Applications, Volume II*, Proceedings of the International Conference and Workshop on Valuation Theory (Saskatoon 1999), F.-V. Kuhlmann, S. Kuhlmann and M. Marshall (editors), The Fields Institute Communications Series **33**, Publications of the American Mathematical Society. With money that we received from the Fields Institute for the camera-ready preparation of this volume, we bought free copies of Vol. I and Vol. II for the authors of the proceedings papers. The work connected with this (preparation of the address lists, negotiating a reduced price with AMS, distribution of the copies) turned out to be much more than expected.