

— Math 872 T2, 2007-08 —

**Graduate course Special Topics in Algebra:
Fields and their closures****COURSE OUTLINE**

This course will give an introduction to an area of field theory that has witnessed particularly interesting applications of model theory. We will discuss various closures of fields: algebraic closure, real closure and p -adic closure. We will study their role in the model theory of fields and model theoretic results about these closures. In particular, we will introduce various Nullstellensätze and demonstrate how they form a model theoretic framework to understand closures of fields. Finally, if time permits, we will give an overview of further closure properties (PAC, PRC, PpC, large fields, etc.).

Here is a tentative list of subjects:

Algebraic closure of fields — orderings — real closures — basic facts about valuations, places and valuation rings — p -adic valuations — Hensel's Lemma — p -adic closures — basic facts from model theory — Nullstellensätze — model theory of algebraically, real and p -adically closed fields — applications.

The students are expected to hand in assignments (about one every other week). A midterm exam will occur in February.

The final mark will be calculated as follows:

Assignments: 25 %
Midterm Exam: 25 %
Final Exam: 50 %.

Some literature (more references will be given during the course):

Alexander Prestel: *Lectures on formally real fields*, Springer Lecture notes in mathematics **1093**, QA3 .L47 no.1093

Alexander Prestel and Peter Roquette: *Formally p -adic fields*, Springer Lecture notes in mathematics **1050**, QA3 .L47 no.1050

Nathan Jacobson: *Lectures in abstract algebra*, Vol. 3, New York : Van Nostrand, The University series in higher mathematics, QA266 .J17 v.3

Antonio J. Engler and Alexander Prestel: *Valued fields*, Springer monographs in mathematics, QA247 .E574 2005