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Form 101 - Application for a Grant

Send to NSERC with your attachments, if applicable

Reference Number: 75270198

Applicant: Murray Bremner **NSERC PIN:** 56851

Program: Discovery Grants - Individual

Application Title: Computational methods in nonassociative algebra

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Expected Paper Attachments:

Murray Bremner

Form 101 - Application for a Grant

- [ ] Research Contribution 1 - DNA computing, insertion of words, and ...
- [ ] Research Contribution 2 - Dimension formulas for the free nonassociative ...
- [ ] Research Contribution 3 - Invariant nonassociative algebra structures on ...
- [ ] Research Contribution 4 - Identities for algebras of matrices over the ...

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FORM 101
Application for a Grant
PART I

Date: 2005/10/28

Family name of applicant	Given name	Initial(s) of all given names	Personal identification no. (PIN)
Bremner	Murray	MR	56851

Institution that will administer the grant	Language of application	Time (in hours per month) to be devoted to the proposed research / activity
Saskatchewan	[X] English [ ] French	80

Type of grant applied for	For Strategic Projects, indicate the Target Area and Sub-Target Area, if applicable.
Discovery Grants - Individual	

Title of proposal
Computational methods in nonassociative algebra

Provide a maximum of 10 key words that describe this proposal. Use commas to separate them.
Nonassociative algebra, Polynomial identities, Computational algebra, Algorithms for linear algebra, Maple programming, Representation theory, Lie algebras, Quantum groups

Research subject code(s) Primary	Research subject code(s) Secondary	Area of application code(s) Primary	Area of application code(s) Secondary
2908	2714	801	1201

CERTIFICATION/REQUIREMENTS

If this proposal involves any of the following, check the box(es) and submit the protocol to the university certification committee.

Research involving : Humans [ ] Human pluripotent stem cells [ ] Animals [ ] Biohazards [ ]

Does any phase of the research described in this proposal a) take place outside an office or laboratory, or b) involve an undertaking as described in Part 1 of Appendix B?

[X] NO [ ] If YES to either question a) or b) – Appendices A and B must be completed

TOTAL AMOUNT REQUESTED FROM NSERC

Year 1	Year 2	Year 3	Year 4	Year 5
45,000	45,000	45,000	45,000	45,000

SIGNATURES (Refer to instructions "What do signatures mean?")

It is agreed that the general conditions governing grants as outlined in the NSERC *Program Guide for Professors* apply to any grant made pursuant to this application and are hereby accepted by the applicant and the applicant's employing institution.

Applicant

Head of department

Dean of faculty

President of university (or representative)

Applicant's department, university, tel. and fax nos., and e-mail
Mathematics and Statistics
Saskatchewan
Tel.: (306) 966-6122
FAX: (306) 966-6086
bremner@math.usask.ca

Personal identification no. (PIN)	Family name of applicant
56851	Bremner

SUMMARY OF PROPOSAL FOR PUBLIC RELEASE (Use plain language.)

This plain language summary will be available to the public if your proposal is funded. Although it is not mandatory, you may choose to include your business telephone number and/or your e-mail address to facilitate contact with the public and the media about your research.

Business telephone no. (optional): (306) 966-6122

E-mail address (optional): bremner@math.usask.ca

The unifying theme of this research program is the developing connections between abstract algebra, computer science, and mathematical genetics.

In abstract algebra, the focus is on using computational methods to study the classical theory of polynomial identities for nonassociative algebras. Most of the classes of algebras studied by mathematicians are defined by identities (such as associative, Lie and Jordan algebras), and many of these structures have applications throughout mathematics and theoretical physics. I am especially interested in the more recent applications to life sciences (in particular, DNA computing and population genetics) and in applications of biological theory to computer science (in particular, genetic algorithms).

My research makes intensive use of computer algebra systems (such as Maple) to implement algorithms from linear algebra, combinatorics, and representation theory (of the symmetric group and of simple Lie algebras).

Second Language Version of Summary (optional).

Personal identification no. (PIN)	Family name of applicant
56851	Bremner

Before completing this section, **read the instructions** and consult the *Use of Grant Funds* section of the NSERC Program Guide for Professors concerning the eligibility of expenditures for the direct costs of research and the regulations governing the use of grant funds.

TOTAL PROPOSED EXPENDITURES (Include cash expenditures only)

	Year 1	Year 2	Year 3	Year 4	Year 5
1) Salaries and benefits					
a) Students	22,500	22,500	22,500	22,500	22,500
b) Postdoctoral fellows	12,000	12,000	12,000	12,000	12,000
c) Technical/professional assistants	0	0	0	0	0
d)	0	0	0	0	0
2) Equipment or facility					
a) Purchase or rental	1,000	1,000	1,000	1,000	1,000
b) Operation and maintenance costs	0	0	0	0	0
c) User fees	0	0	0	0	0
3) Materials and supplies	500	500	500	500	500
4) Travel					
a) Conferences	4,000	4,000	4,000	4,000	4,000
b) Field work	0	0	0	0	0
c) Collaboration/consultation	5,000	5,000	5,000	5,000	5,000
5) Dissemination costs					
a) Publication costs	0	0	0	0	0
b)	0	0	0	0	0
6) Other (specify)					
a)	0	0	0	0	0
b)	0	0	0	0	0
TOTAL PROPOSED EXPENDITURES	45,000	45,000	45,000	45,000	45,000
Total cash contribution from industry (if applicable)					
Total cash contribution from university (if applicable)					
Total cash contribution from other sources (if applicable)	0	0	0	0	0
TOTAL AMOUNT REQUESTED FROM NSERC (transfer to page 1)	45,000	45,000	45,000	45,000	45,000

Budget justification

1 Salaries and benefits

1.1 Students

- \$2,000 per year for a summer undergraduate research student (on top of USRA funds from NSERC).

- \$20,500 per year for a Ph.D. student: Basic stipend of \$19,000 per year, plus non-discretionary benefits with a multiplier of 8.34% (allowing the student to take 3 weeks of paid vacation per year following the Saskatchewan labour standard), for a total of \$20,584.60. I was contacted almost one year ago by a potential Ph.D. student (**Hader Elgendy**, whose husband is currently doing graduate studies in Geology at the University of Alberta). On my suggestion, she has already done substantial background reading on a topic in my current research proposal: nonassociative structures on representations of simple Lie algebras.

1.2 Postdoctoral fellows

- \$12,000 per year: Salary of \$30,000 per year (including benefits) for two post-doctoral fellowships, each of 12 months' duration, during the five years of the grant, for a total of \$60,000 (averaged over five years). Each PDF would be spending 80–100% of his/her time on this research project. The responsibilities will include theoretical research on one or more of the projects in the grant proposal, computer programming in Maple and C, and regular participation in seminars (algebra, bioinformatics, computer science). There are active research groups in nonassociative algebra in Spain and Latin America, and I expect to be able to recruit post-doctoral fellows from my colleagues in those countries.

2 Equipment or facility

2.1 Purchase or rental

- \$1,000 per year: I will need to acquire a new desktop computer within the next five years. I expect that this will cost roughly \$5,000 (machine, upgrades, software), averaged over five years. One possibility (recommended by our systems manager R. Kondra) is an HP xw6200 Workstation with dual 3.2 GHz processors and 3 GB RAM together with two 20" monitors; a price quote (25 October 2005) for this system is \$5,047 plus tax. For larger Maple jobs I may need to increase this to 5 GB RAM, which would add about \$1,000 to the total cost.

3 Materials and supplies

- \$500 per year: photocopies, faxes, phone calls, and possibly books which are not available in the university library or through interlibrary loan.

4 Travel

4.1 Conferences

- $4,000 per year: This will permit me to travel to one or two conferences per year, including:
  - August 2006: ICM in Madrid (22–30 August) preceded by satellite conference in Oviedo on Associative and Non-associative Algebraic Structures and Applications (18–20 August).
  - August 2007: Week-long Conference on Nonassociative Algebra in honour of the 60th birthday of Ivan Shestakov at the University of São Paulo.
  - January 2008: I expect to have a sabbatical leave during the first six months of 2008. During my last leave (January to June 2004) I organized, together with Hentzel and Peresi, a Special Session on Nonassociative Algebra at the January meeting of the American Mathematical Society in Phoenix. I plan to apply for another Special Session at the January 2008 meeting in San Diego.
  - Third Conference on Lie and Jordan Algebras, their Representations and Applications, in Guarujá, Brazil, in either 2008 or 2009. (The first two were in 2002 and 2004.)
  - Sixth International Conference on Nonassociative Algebra and its Applications. The last conference in this sequence was held in Mexico in 2003.
  - Maple Conference in Waterloo, held annually since 2002 (the 2003 conference was cancelled because of SARS).
  - ISSAC (International Symposium on Symbolic and Algebraic Computation) meets every year, alternating between North America and Europe (or Asia).
  - There are regular international meetings in the areas of (1) DNA computing, (2) evolutionary computation and (3) population genetics. I plan to attend at least one of these conferences during the next five years.

4.2 Collaboration

- $5,000 per year: This will fund a two-week visit twice each year (February, May or August) to the University of São Paulo for joint work with Luiz A. Peresi; I have made four research trips to USP in the last three years. Round-trip airfare from Saskatoon to São Paulo is currently about $1,600. Accommodation for two weeks in São Paulo currently costs about $840. I have found that my productivity increases substantially when I collaborate. Peresi has substantial background in general nonassociative algebra and particular expertise in genetic algebras; he is also an expert in using the C code written by Hentzel to apply representation theory of the symmetric group to the study of polynomial identities. At the University of São Paulo, there is a very strong research group in nonassociative algebra, which includes the famous Russian mathematician I. P. Shestakov (co-author of the standard reference on nonassociative algebra), the Spanish mathematician J. C. Gutiérrez (who completed the classification of Bernstein algebras), the German applied mathematician F. M. Forger (who has applied the representation theory of Lie algebras to the study of symmetries in the genetic code), and the Brazilian mathematician H. Guzzo (who works on genetic algebras and generalizations).

Relationship to other support

I do not have any other funding except for my current NSERC research grant, and a portion of my salary from the University of Saskatchewan which I have taken as a research grant (Research Grant in Lieu of Salary, and Sabbatical Leave Research Grant in Lieu of Salary). I applied for the latter funding from the U. of S. when my NSERC balance was negative two years ago.

1 Overview of proposed research

The unifying theme of this research program is the connections between abstract algebra, computer science, and mathematical genetics. In abstract algebra, the focus is on polynomial identities for nonassociative algebras, and the closely related topic of free algebras. Most of the well-known classes of algebras, such as associative, alternative, Lie and Jordan algebras, are defined in terms of identities. In computer science, the focus is on using computer algebra systems to find identities for algebras; the algorithms come from linear algebra, combinatorics and representation theory. In mathematical genetics, there are three topics: (1) the study of bio-operations arising in DNA computing; (2) the application of genetic algorithms to problems in abstract algebra; (3) the theory of Bernstein algebras, and its connections with population genetics.

2 Recent progress in research related to the proposal

I have been developing computational techniques for finding polynomial identities for the past ten years, with a focus on structures and operations which satisfy previously unknown identities. (1) For DNA computing, the results are [18] (for reference numbers ≤ 22 see Form 100) and recently submitted work [17] on polynomial identities for intermolecular recombination. (2) For genetic algorithms, a Maple program I wrote recently has already simplified the results of work in progress with Peresi on the classification of trilinear operations and their minimal identities. (3) For Bernstein algebras, the results are work in progress with my summer 2005 undergraduate research student **S. Richards**; during summer 2005 I also prepared detailed notes on the problem of nilpotence of the ideal of weight zero elements in a Bernstein algebra. (4) For nonassociative structures on representations of simple Lie algebras, the results are [6], [19]. (5) For free nonassociative algebras, the results are [3], [4], [5]. (6) For ternary structures, the results are [2] and the work in progress with Peresi on trilinear operations. (7) For alternative algebras, the results are a simplification using a genetic algorithm of the degree 8 identities discovered in [7].

3 Objectives: both short- and long-term

The short-term objectives are completion and submission of work in progress on (1) polynomial identities for higher-order Bernstein algebras (joint with my 2005 summer student **S. Richards**), (2) genetic algorithms and their application to polynomial identities, (3) classification of trilinear operations and their minimal identities (joint with Peresi), (4) higher identities for the algebra of 2×2 matrices over the octonions (joint with Hentzel). Two medium-term objectives are (5) completion and submission of the BIRS project on enveloping algebras of Malcev algebras (joint with Hentzel and Peresi), (6) submission of a proposal to Waterloo Maple Inc. for software to perform computations on nonassociative algebras (an invitation was received in May 2005). The long-term objectives are to expand a vigorous research program connecting abstract algebra, computer science, and mathematical genetics, and to involve more students (at all levels) and co-authors. Particular aspects of this program are described in the following two subsections.

3.1 Topics in mathematical genetics

DNA computing: The theory of DNA computing studies properties of bio-operations on formal languages which mimic the processes of molecular genetics. Formal languages are subsets of free monoids, and so these operations are usually defined on sets of words. In the last two years I have approached this topic from the point of view of nonassociative algebra. One specifies a field and regards a finite set of words as the linear combination of its members. This linearization permits the study of algebraic properties of operations, such as polynomial identities, which are difficult to describe in terms of formal languages. It also permits a natural representation of multisets and probability distributions: the coefficient of a word is its multiplicity or probability. Using my computational methods, I have found that familiar operations from DNA computing give rise to natural nonassociative structures: insertion of words [25] satisfies the left-symmetric identity [18] and intermolecular recombination [29] satisfies an identity which implies the Jordan identity [17]. I plan to use the methods described in Section 5 to extend this work to the many other bio-operations studied in the literature of DNA computing.

Evolutionary computation: Genetic algorithms have been used since the 1970s as a powerful tool in combinatorial optimization. I have recently written (and implemented in Maple) an algorithm to search for an optimal basis for the nullspace of a matrix with rational entries: a basis consisting of vectors, each of which has integral components with no common factor, such that the maximum component is as small as possible. The basic idea is to randomly permute the columns (giving a different set of free variables), recompute the row canonical form to obtain a new basis of the nullspace, and then use a selection procedure to derive a better basis from the old and new bases. In nonassociative algebra, this algorithm can be used to produce polynomial identities which are shorter and simpler than those previously known: it has already found smaller identities for the octonion matrix algebra studied in [7] and for the trilinear operations which appear in current joint work with Peresi. I plan to study further applications of this algorithm and other methods of evolutionary computation to problems in the theory of polynomial identities. (This topic, unlike the other two in this subsection, is an application of biological ideas to mathematical problems: biological mathematics rather than mathematical biology).

Population genetics: This is the oldest and most substantial application of mathematics to biology. The classical tradition in this area uses the theory of stochastic processes [26], [36]. A subsidiary tradition, originating with Etherington and developed by Holgate and Lyubich, applies the theory of nonassociative algebras [31]. The most important class of algebras in Lyubich's approach is the (first-order) Bernstein algebras; their definition encapsulates, in the form of a polynomial identity, the Hardy-Weinberg principle that gene frequencies in a population stabilize after a single generation of random mating. These two approaches have points of contact: see [26] pp. 208, 216, 250 and [36] p. 233. Lyubich classified the Bernstein algebras which are most relevant to genetics, but the complete classification from an algebraic point of view was given recently by Gutiérrez [28]. I plan to apply my techniques of computational search for polynomial identities to Bernstein algebras, and in particular to n-th order Bernstein algebras (which represent populations whose gene frequencies stabilize after n generations): these higher-order algebras have received little attention. Together with my summer 2005 undergraduate research student **S. Richards**, I have used computer algebra to discover new polynomial identities satisfied by the second-, third- and fourth-order Bernstein algebras of simple Mendelian inheritance. Our most interesting result is that even though the first-order algebras on two and three alleles *cannot* be distinguished by their polynomial identities, the higher-order algebras *can*. I plan also to work on two other closely related problems on higher-order Bernstein algebras: (1) determination of conditions for an algebra to be a genetic algebra, and (2) determination of an upper bound for the index of nilpotence of the ideal of weight zero elements.

3.2 Topics in abstract algebra

Enveloping algebras for Malcev algebras (with Hentzel and Peresi): We started this project at BIRS in May 2005. The inspiration comes from recent work of Pérez-Izquierdo and Shestakov [33], who used Lie triple systems to extend the theorem of Poincaré, Birkhoff and Witt to Malcev algebras. We plan to use computer algebra to determine explicit structure constants for the universal enveloping algebra of the non-Lie Malcev algebra of dimension 4. With these we can determine the polynomial identities and the largest alternative quotient. Beyond this, there are many non-Lie Malcev algebras in dimension 5: one nilpotent, one non-solvable, and an infinite family of solvable non-nilpotent algebras [30]. This work leads up to the most important case: the simple non-Lie Malcev algebra of dimension 7. These enveloping algebras are basic examples of nonassociative Hopf algebras. The ultimate goal is to settle the speciality question: Is every Malcev algebra isomorphic to a subalgebra of the commutator algebra of an alternative algebra?

Nonassociative structures on representations of Lie algebras (with Hentzel, and possibly a new graduate student) [6], [19]: Using our computer algebra techniques, Hentzel and I determined the minimal polynomial identities for the binary and ternary alternating nonassociative structures on certain irreducible representations of the Lie algebra $sl(2)$ which admit a unique such structure. I plan to extend these results in two different ways: (1) For a representation admitting two linearly independent structures, I will use the Smith normal form of a matrix over a polynomial ring to determine parameter values which give algebra structures satisfying an identity of low degree; this technique has been used successfully in a different context in recent work with Peresi on trilinear operations. (2) I will study nonassociative structures on representations of simple Lie algebras of rank ≥ 2; in this case, the computer algebra system `LiE` [37] can be used to determine the decomposition of the exterior powers of a representation. I have been contacted by a prospective graduate student (**Hader Elgendy**) who is doing background reading on this project.

Dimension formulas for free nonassociative algebras (with Hentzel and Peresi) [3], [4]: Inspired by recent work of Shestakov and Umirbaev [35], we discovered a generalization of the Witt dimension formula for free Lie algebras to the case of free Akivis algebras. Since in general exact dimension formulas are hard to obtain for nonassociative algebras, this is a promising direction for further research. Our results involve a new integer sequence of combinatorial interest, the log-Catalan numbers, and so this work has the potential for connections with algebraic combinatorics. Related work has been done using the theory of operads by Gerritzen and Holtkamp [27]. I plan to apply these combinatorial methods for dimension formulas to other varieties of nonassociative algebras. This project connects my computational methods with mainstream research in the general theory of polynomial identities.

Classification of simple Lie coloralgebras (with **Kochetov**): Recent results by Bahturin and his co-authors [23] on gradings for simple Lie algebras make it possible to undertake the classification of simple Lie coloralgebras with small grading groups. The results are well-known for Lie algebras (group of order 1) and Lie superalgebras (group of order 2). **Kochetov** and I plan to classify the algebras arising from grading groups of orders 3 and 4. These algebras will provide new types of symmetry with applications in the natural sciences. (**Kochetov** and I have previously collaborated on a translation of the *Dniester Notebook* [20], an important Russian collection of unsolved problems in algebra.)

Quantization of nonassociative structures: In my previous grant application I mentioned the possibility of extending my work [13] on the octonions to the quantization of the simple exceptional Jordan algebra. This will involve representations of the quantum group of type F_4, since the corresponding Lie algebra is the derivation algebra. I am looking for a collaborator for this project.

Polynomial identities for relation algebras in characteristic 0 (with **El Bachraoui**): In 2003–2004 I supervised a post-doctoral fellow with a Ph.D. in algebraic logic (relation algebras) from the Free University of Amsterdam. Our current project is a computer search for polynomial identities satisfied by relation algebras in characteristic 0 involving both binary and higher-order relations (unlike the classical theory which considers only characteristic 2 and includes only binary relations).

4 Literature pertinent to the proposal

The basic general reference for nonassociative algebras and their polynomial identities is [40]. For computer algebra see [38]; an important paper for computer implementations of the representation theory of the symmetric group is [24]. For DNA computing see [32] and the papers [25], [29]. For genetic algorithms see [34]. The classical theory of mathematical genetics is presented in [26], [36]. For applications of nonassociative algebra in this area see [31], [39] and for the classification of first-order Bernstein algebras see [28]. Further references have been mentioned in appropriate places in other parts of this proposal.

5 Methods and proposed approach

The primary method in my study of polynomial identities for nonassociative structures is computational search techniques implemented with computer algebra systems (primarily Maple). I use exhaustive searches involving linear algebra on large matrices, together with the representation theory of the symmetric group, to decompose multilinear identities into irreducible pieces. These techniques make it possible to discover identities which would be unattainable by hand calculation, although once the identities are known a direct proof is sometimes possible. In the last few months I have started to apply genetic algorithms to find identities which are equivalent to, but better than, previously known identities (fewer terms and smaller coefficients).

Over a field of characteristic 0 (or p greater than the degree of the identity) any identity is equivalent to a set of multilinear identities. The basic method for finding identities is to construct a large matrix, called the expansion matrix: the columns of this matrix correspond to the inequivalent nonassociative monomials. Each column contains the expansion of the corresponding monomial using the given algebra or operation. The nonzero elements of the nullspace of the expansion matrix represent polynomial identities satisfied by the structure. These expansion matrices typically have hundreds or thousands of rows and columns, and so it is important to have efficient algorithms to process them. One usually prefers to do these computations over the rational numbers, but since the matrix entries can become extremely large at an intermediate stage (numerators and denominators with hundreds of decimal digits), it is often necessary to do the calculations over a finite field to control memory allocation, and then relate the modular results to the rational case.

This algorithm can be refined using the representation theory of the symmetric group S_n. The vector space of all multilinear nonassociative polynomials of degree n naturally has the structure of an S_n-module. The subspace of identities satisfied by a given structure is a submodule, and it decomposes into a direct sum of irreducible components. These components are vector spaces of much smaller dimension than those arising from the method described in the previous paragraph. This allows a more efficient solution of a given problem, and the extension of the search to higher degrees. For a detailed discussion and examples see my papers with Hentzel [6], [7].

6 Anticipated significance of the work

This is threefold: (1) Development of the theory of polynomial identities for nonassociative algebras and the structure theory of free nonassociative algebras. (2) Development of efficient and effective algorithms for problems in abstract algebra, and their implementation in computer algebra systems. (3) Development of connections (in both directions) between abstract algebra and mathematical genetics in order to promote the cross-fertilization of these areas.

7 Training to take place through the proposal

Many of the projects described in this proposal are appropriate for students at all levels: undergraduate, graduate, post-doctoral. Projects involving computer algebra, mathematical genetics, and representation theory will make a direct contribution to the scientific and technological agenda of the federal granting agencies. The first two of these areas are especially relevant because of the numerous employment opportunities.

As a result of administrative problems in my Department, we have not been allowed to admit new graduate students (except in special cases) for the last few years. There are signs that this situation will be resolved in the near future: we have been authorized to initiate a search for a new external Head. Because of this situation, for the last few years I have concentrated on supervising summer undergraduate students and collaborating with post-doctoral fellows.

8 Relevance to the most recent Reallocations Exercise

The document *A Vision for Mathematics in Canada* by the Pure and Applied Mathematics Steering Committee emphasized the recent expansion of Canadian mathematics into applications such as the life sciences and information technology; my current research program is closely related to both of these developments. "Algebraists are typically involved in, and motivated by, problems from other areas of mathematics or theoretical physics" (p. 10); one should add biology and computer science to this statement, given the important applications of abstract algebra to mathematical genetics and bioinformatics. The document also mentioned as priorities the strengthening of leadership in pure and applied research, the training of HQP, and the development of research groups. The current proposal has both pure and applied aspects, and involves joint work with PDFs (**Kochetov**: Russia, **El Bachraoui**: Morocco) and co-authors (Hentzel: USA, Peresi: Brazil). However, the document points out that "current funding levels in mathematics are too low to adequately support the necessary numbers of HQP ... research grants are a necessary and major component of HQP support" (p. 7), and "recruitment of PDFs is a priority in Canadian mathematics" (p. 23). The response of the Reallocations Committee to this document emphasized the strong interdisciplinary nature of Canadian mathematics, in particular computational mathematics and mathematical biology, two of the main areas in the current proposal.

[23] Y. A. Bahturin, I. P. Shestakov and M. V. Zaicev, *Gradings on simple Jordan and Lie algebras*, Journal of Algebra 283 (2005) 849–868.

[24] J. Clifton, *A simplification of the computation of the natural representation of the symmetric group S_n*, Proceedings of the American Mathematical Society 83 (1981) 248–250.

[25] M. Daley, Lila Kari and I. McQuillan, *Families of languages defined by ciliate bio-operations*, Theoretical Computer Science 320 (2004) 51–69.

[26] W. J. Ewens, *Mathematical Population Genetics I: Theoretical Introduction*, second edition, Springer-Verlag, New York, 2004.

[27] L. Gerritzen and R. Holtkamp, *Hopf co-addition for free magma algebras and the non-associative Hausdorff series*, Journal of Algebra 265 (2003) 264–284.

[28] J. C. Gutiérrez, *Solution of the Bernstein problem in the non-regular case*, Journal of Algebra 223 (2000) 109–132.

[29] Lila Kari and Laura Landweber, *The evolution of cellular computing: nature's solution to a computational problem*, Biosystems 52 (1999) 3–13.

[30] E. N. Kuzmin, *Malcev algebras of dimension five over a field of characteristic zero*, Algebra and Logic 9 (1970) 691–700.

[31] Y. I. Lyubich, *Mathematical Structures in Population Genetics*, translated from the 1983 Russian original by D. Vulis and A. Karpov, Springer-Verlag, Berlin, 1992.

[32] G. Paŭn, G. Rozenberg and A. Salomaa, *DNA Computing: New Computing Paradigms*, Springer-Verlag, Berlin, 1998.

[33] J. M. Pérez-Izquierdo and I. P. Shestakov, *An envelope for Malcev algebras*, Journal of Algebra 272 (2004) 379–393.

[34] C. R. Reeves and J. E. Rowe, *Genetic Algorithms: Principles and Perspectives. A Guide to GA Theory*, Kluwer Academic Publishers, Boston, 2003.

[35] I. P. Shestakov and U. U. Umirbaev. *Free Akivis algebras, primitive elements, and hyperalgebras*, Journal of Algebra 250 (2002) 533–548.

[36] Y. M. Svirezhev and V. P. Passekov, *Fundamentals of Mathematical Evolutionary Genetics*, Kluwer Academic Publishers, Dordrecht, 1990.

[37] M. A. A. van Leeuwen, *LiE: A software package for Lie group computations*, Euromath Bulletin 1 (1994) 83–94.

[38] J. von zur Gathen and J. Gerhard, *Modern Computer Algebra*, second edition, Cambridge University Press, Cambridge, 2003.

[39] Angelika Wörz-Busekros, *Algebras in Genetics*, Springer-Verlag, Berlin, 1980.

[40] K. A. Zhevlakov, A. M. Slinko, I. P. Shestakov and A. I. Shirshov, *Rings that are Nearly Associative*, translated from the Russian by Harry F. Smith, Academic Press, New York, 1982.

Murray Bremner

Form 101 - Application for a Grant

Research Contribution 1

DNA computing, insertion of words, and ...

Murray Bremner

Form 101 - Application for a Grant

Research Contribution 2

Dimension formulas for the free nonassociative ...

Murray Bremner

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Research Contribution 3

Invariant nonassociative algebra structures on ...

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Research Contribution 4

Identities for algebras of matrices over the ...

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Investing in people, discovery and innovation
Investir dans les gens, la découverte et l'innovation

FORM 100
Personal Data Form
PART I

Date: 2005/10/28

Family name	Given name	Initial(s) of all given names	Personal identification no. (PIN)
Bremner	Murray	MR	56851

- [X] I hold a full, an associate or an assistant professor position at a Canadian university
- [ ] I do not or will not hold an academic appointment at a Canadian postsecondary institution
- [ ] I hold an academic appointment at a Canadian university but am not a full, an associate or an assistant professor (complete Appendices B and C)
- [ ] I hold a faculty position at an eligible Canadian college (complete Appendices B1 and C)

Place of employment other than a Canadian postsecondary institution (give address in Appendix A)

APPOINTMENT AT A POSTSECONDARY INSTITUTION

Title of position	Canadian postsecondary institution
Professor	Saskatchewan
Department	**Campus**
Mathematics and Statistics	

ACADEMIC BACKGROUND

Degree	Name of discipline	Institution	Country	Date yyyy/mm
Bachelor's	Mathematics and Far Eastern Studies	Saskatchewan	CANADA	1981 / 05
Master's	Computer Science	Concordia	CANADA	1984 / 05
Doctorate	Mathematics	Yale University	UNITED STATES	1989 / 05

TRAINING OF HIGHLY QUALIFIED PERSONNEL

Indicate the number of students, fellows and other research personnel that you:

	Currently		Over the past six years (excluding the current year)		
	Supervised	Co-supervised	Supervised	Co-supervised	Total
Undergraduate	1		3	1	5
Master's				1	1
Doctoral		1			1
Postdoctoral			1	1	2
Others					
Total	1	1	4	3	9

Personal identification no. (PIN)	Family name
56851	Bremner

ACADEMIC, RESEARCH AND INDUSTRIAL EXPERIENCE (use one additional page if necessary)

Position held (begin with current)	Organization	Department	Period (yyyy/mm to yyyy/mm)
Professor	Saskatchewan	Mathematics and Statistics	2002/07
Associate Professor	University of Saskatchewan	Mathematics and Statistics	1995/07 to 2002/06
Assistant Professor	University of Saskatchewan	Mathematics and Statistics	1993/07 to 1995/06
Assistant Professor	University of Toronto	Mathematics	1990/07 to 1993/06
Postdoctoral Fellow	MSRI, Berkeley	-	1989/07 to 1990/06

Personal identification no. (PIN)	Family name
56851	Bremner

RESEARCH SUPPORT

Family name and initial(s) of applicant	Title of proposal, funding source and program, and time commitment (hours/month)	Amount per year	Years of tenure (yyyy)
List all sources of support **(including NSERC grants and university start-up funds)** held as an applicant or a co-applicant: a) support held in the past four (4) years but now completed; b) support currently held, and c) support applied for. For group grants, indicate the percentage of the funding directly applicable to your research. Use additional pages as required.			
a) Support held in the past 4 years			
Murray R. Bremner	Nonassociative algebra University of Saskatchewan Research grant in lieu of salary 80 hours/month	8,750 8,700	2004 2005
b) Support currently held			
Murray R. Bremner	Nonassociative algebra NSERC Research grant 80 hours/month	10,000 10,000 10,000 10,000 10,000	2001 2002 2003 2004 2005
c) Support applied for			
Murray R. Bremner	Computational aspects of nonassociative algebra NSERC Discovery Grant 80 hours/month	45,000 45,000 45,000 45,000 45,000	2006 2007 2008 2009 2010

Highly Qualified Personnel (HQP)

Provide personal data about the HQP that you currently, or over the past six years, have supervised or co-supervised.

Personal identification no. (PIN)	Family name
56851	Bremner

Name	Type of HQP Training and Status	Years Supervised or Co-supervised	Title of Project or Thesis	Present Position
Richards, Sheldon	Undergraduate (In Progress)	Supervised 2005 - 2005	Polynomial identites for genetic algebras	Undergraduate student, Comp. Sci. and Math., Saskatchewan
Lataianu, Bogdan	Doctoral (In Progress)	Co-supervised 2003 - 2005	Extensions of valuations and key polynomials	Ph.D. student, Mathematics, Saskatchewan
El Bachraoui, Mohamed	Postdoctoral (Completed)	Supervised 2003 - 2004	Relation algebras and nonassociative algebras	Assistant Professor, Al-Akhawayn University, Morocco
Bains, Lauren	Undergraduate (Completed)	Supervised 2003 - 2003	Invariant nonassociative algebra structures	M.Sc. student, Physics, UBC
Li, Ying	Undergraduate (Completed)	Supervised 2003 - 2003	Invariant nonassociative algebra structures	M.Sc. student, Bioinformatics, Saskatchewan
Kochetov, Mikhail	Postdoctoral (Completed)	Co-supervised 2002 - 2003	Nonassociative coalgebras	Assistant Professor, DePaul University, Chicago
Mihilewicz, Kris	Undergraduate (Completed)	Supervised 2002 - 2002	Mathematical theory of music	M.Sc. student, Physics, Saskatchewan
(Name withheld)	Master's (Completed)	Supervised 2001 - 2002	(continued under another supervisor)	Unknown
Lee, Jonathan	Undergraduate (Completed)	Co-supervised 2001 - 2001	Computer algebra in free nonassociative algebras	Ph.D. student, Mathematics, Stanford University

1 Most significant contributions to research

1.1 DNA computing [18]

The abstract algebraic branches of bioinformatics study molecular genetics from the point of view of theoretical computer science; the goal is to discover how to use molecular (rather than electronic) processes as basic computer hardware. At present, this theory uses the concepts of formal languages; my new approach is to reinterpret the language operations as nonassociative algebra products, and study their polynomial identities. Paper [18] does this for normal and synchronized insertion on words (as studied in recent work of Daley, Kari and McQuillan, see Form 101 for the reference). I show that the linearization of normal insertion satisfies the left-symmetric identity in degree 3. Building on recent work of Segal (which provides a basis for free left-symmetric algebras) I showed that every identity of degree ≤ 5 for normal insertion follows from the left-symmetric identity. I further showed that synchronized insertion does not satisfy *any* polynomial identity in degree ≤ 5, even over an alphabet with only one element.

1.2 Representations of Lie algebras [6], [19]

In these papers Hentzel and I use the representation theory of the simple Lie algebra $sl(2)$ (in particular, an explicit form of the Clebsch-Gordan theorem) to determine which irreducible representations $V(n)$ occur exactly once in their own exterior squares or cubes. These representations admit unique nonassociative alternating binary or ternary structures which are invariant in the sense that they have $sl(2)$ in their derivation algebras. For $V(10)$ we explicitly determined the structure constants of the resulting 11-dimensional nonassociative algebra and its minimal identities (which have degree 7). This work was referred to in the recent paper by Benito, Draper and Elduque, *Lie-Yamaguti algebras related to* G_2, J. Pure Appl. Algebra 202 (2005) 22–54. For $V(n)$ with $n = 3, 5, 6, 7, 8, 10$ we determined the structure constants of the ternary algebras and their minimal identities. This work can be extended in many directions: bigger representations, higher exterior powers, other Lie algebras. The original idea for this project was mine, and the necessary representation theory of $sl(2)$ was worked out by me; Hentzel and I collaborated on the computational implementation. My 2003 summer undergraduate research student (**Lauren Bains**) made a significant contribution to paper [6].

1.3 Free nonassociative algebras [3], [4], [5]

The first two papers are joint work with Hentzel and Peresi; the third is joint work with Hentzel. Papers [3], [4] prove a generalization of the famous Witt dimension formula for free Lie algebras. One can interpret the Witt formula in terms of the free associative algebra: it gives the dimension of the space of primitive elements (in the sense of Hopf algebras) in each degree. Our formula does the same for the free nonassociative algebra; it introduces a new integer sequence of combinatorial interest, the log-Catalan numbers. This result is based on earlier work of Shestakov and Umirbaev, and is closely related to recent work of R. Holtkamp on nonassociative Hopf algebras (*On Hopf algebra structures over free operads*, math.RA/0504531) and of Pérez-Izquierdo on Sabinin algebras (*Algebras, hyperalgebras, nonassociative bialgebras and loops*, to appear in Advances in Mathematics). The original idea for this project was mine, and I explained the background material on free Lie algebras to Hentzel and Peresi; the three of us collaborated on the computational implementation.

Paper [5] was inspired by the definition of Akivis algebras in terms of the polynomial identities relating the Lie bracket and the associator in the free nonassociative algebra. It studies a related problem: determining the identities relating the Jordan product and the associator. We discovered a complicated collection of identities, which contrasts strikingly with the Akivis case. The original idea for this project was mine; Hentzel and I collaborated on the computational implementation.

1.4 Ternary algebras [2], [10], [11], [12], [15]

Paper [11] is joint work with Hentzel, and paper [2] is joint work with Peresi. One of my ongoing projects is the problem of finding the "correct" generalization of Lie, Jordan and Malcev algebras to the ternary case. Work by Loday and his students on the theory of operads has shown that for n-ary algebras with $n \geq 3$ there are two inequivalent definitions of associativity: total and partial associativity. Both definitions reduce to the familiar identity $(ab)c - a(bc) = 0$ in the binary case. Paper [12] studies the $\mathbb{Z}$-module structure of the free partially associative ternary ring on one free generator: it shows that this ring has additive torsion, unlike the free totally associative ternary ring. Papers [10] and [15] led to my joint work with Hentzel [11]: these papers determine the minimal polynomial identities (non-trivial identities of lowest degree) for the alternating sum (generalizing the Lie bracket), the symmetric sum (generalizing the Jordan product), and the cyclic sum (which has no analogue in the binary case). These identities define new varieties of nonassociative ternary algebras analogous to Lie and Jordan algebras. The original idea for this project was mine, and using my computational techniques I could study identities up to degree 7; the collaboration with Hentzel using his methods allowed us to go up to degree 9. A conjecture from my earlier work in this area [16] has recently been proved by M. Rotkiewicz (*On strong n-Lie-Poisson algebras*, J. Pure Appl. Algebra 200 (2005) 87–96; see also *Irreducible identities of n-algebras*, Acta Math. Univ. Comenian. (N.S.) 72 (2003) 23–44). Paper [2] with Peresi extends this work by using the total and partial associators to define generalizations of alternative algebras to the ternary case. We then determine the minimal identities for the alternating sum in these algebras to obtain a generalization of Malcev algebras. The original idea for this project was mine; Peresi and I used computer programs implementing the representation theory of the symmetric group written in Maple by myself, and in C by Hentzel. Further joint work in progress with Peresi gives a complete classification of trilinear operations and determines their identities of degrees ≤ 5. This work suggests a natural quantization of the Lie and Jordan triple products, and introduces some new ternary operations which have not been studied before.

1.5 Alternative algebras [1], [7], [8], [9], [13], [14]

All of these papers study generalizations of alternative algebras; papers [7], [8], [9], are joint work with Hentzel. Paper [9] studies the (binary) algebras obtained by extending the Cayley-Dickson doubling process beyond the octonions to a 16-dimensional algebra (the"sedenions"). We determined the minimal identities satisfied by this algebra, and showed that all the subsequent algebras obtained by the doubling process satisfy the same identities. The original idea for this project was mine; Hentzel and I collaborated on the computational implementation. Paper [8] determines the minimal identities satisfied by the associator $(ab)c - a(bc)$ in three different alternative algebras: the free alternative algebra, the octonion algebra, and a 4-dimension subalgebra of the octonions which is isomorphic to the ternary cross product. These identities can be regarded as elements of the free alternative algebra and are related to ongoing research on the radical, nucleus and centre of free alternative algebras. The original idea for this project was mine, but Hentzel has more background in the study of free alternative algebras; Hentzel and I collaborated on the computational implementation. For recent work on this problem see Hentzel and Peresi, *A nonzero element of degree 7 in the center of the free alternative algebra*, Comm. Algebra 31 (2003) 1279–1299. Paper [7] determines the minimal identities satisfied by the algebra of 2×2 matrices over the octonions. The minimal identities have degree 7; after discovering them with computer programs we wrote in Maple and C, we were able to prove them directly. We also determined that this algebra satisfies further identities in degree 8 which do not follow from the identities in degree 7. (We have recently used a genetic algorithm to simplify these identities in degree 8; we plan to submit a second paper on this algebra within the next year or two.) The original idea for this project was mine; Hentzel and I collaborated on the computational implementation. Paper [14] studies the Abelian group structure of the free semialternative (assocyclic) ring on one generator: this provides an elementary example of a variety of nonassociative rings in which the free rings have additive torsion. Paper [1] gives a detailed example of a power-associative algebra satisfying the unusual property that when the scalars are extended to a larger field, the resulting algebra fails to be power-associative. Paper [13] uses the representation theory of the quantum group corresponding to the simple Lie algebra $sl(2)$ to construct a "quantization" of the octonion algebra. A natural extension of this work would be to study the structure of this algebra as a module over the quantum groups of types G_2 and D_4.

2 Research contributions

See the previous section and the list of references below. A contribution which is not original research, but which will benefit the international community of researchers in nonassociative algebra, is my translation (joint work with **Kochetov**, a former post-doctoral fellow) of the *Dniester Notebook* [20], an important Russian collection of unsolved research problems in abstract algebra. Another translation from Russian (joint work with **Natalia Fomenko**, a former graduate student in my Department) is of the famous 1958 survey article by Shirshov on rings that are nearly associative [21]. Neither of these works was previously available in English translation. *For every contribution the primary source of funding is NSERC.*

References

Articles in refereed publications: published or accepted

[1] M. R. Bremner, *An algebra which is power associative but not strictly power associative*, 4 pages, accepted 18 October 2005, to appear in Communications in Algebra.

[2] M. R. Bremner and L. A. Peresi, *Ternary analogues of Lie and Malcev algebras*, 19 pages, accepted 11 September 2005, to appear in Linear Algebra and its Applications.

[3] M. R. Bremner, I. R. Hentzel and L. A. Peresi, *Dimension formulas for the free nonassociative algebra*, 19 pages, accepted 11 May 2005, to appear in Communications in Algebra.

[4] M. R. Bremner, I. R. Hentzel and L. A. Peresi, *Dimension formulas for the free nonassociative algebra*, Lecture notes from my presentation at the Second Conference on Lie and Jordan Algebras, their Representations and Applications (Guarujá, Brazil, 3–8 May 2004), Resenhas

do Instituto de Matemática e Estatística da Universidade de São Paulo, volume 6, numbers 2/3 (2004), pages 141–151.

[5] M. R. Bremner and I. R. Hentzel, *Identities relating the Jordan product and the associator in the free nonassociative algebra*, 13 pages, accepted 15 November 2004, to appear in Journal of Algebra and its Applications.

[6] M. R. Bremner and I. R. Hentzel, *Invariant nonassociative algebra structures on irreducible representations of simple Lie algebras*, Experimental Mathematics 13 (2004) 231–256.

[7] M. R. Bremner and I. R. Hentzel, *Identities for algebras of matrices over the octonions*, Journal of Algebra 277 (2004) 73–95.

[8] M. R. Bremner and I. R. Hentzel, *Identities for the associator in alternative algebras*, Journal of Symbolic Computation 33 (2002) 255–273.

[9] M. R. Bremner and I. R. Hentzel, *Identities for algebras obtained from the Cayley-Dickson process*, Communications in Algebra 29 (2001) 3523–3534.

[10] M. R. Bremner, *New ternary versions of Jordan algebras*, Algebra Colloquium 8 (2001) 11–24.

[11] M. R. Bremner and I. R. Hentzel, *Identities for generalized Lie and Jordan products on totally associative triple systems*, Journal of Algebra 231 (2000) 387–405.

[12] M. R. Bremner, *On free partially associative triple systems*, Communications in Algebra 28 (2000) 2131–2145.

[13] M. R. Bremner, *Quantum octonions*, Communications in Algebra 27 (1999) 2809–2831.

[14] M. R. Bremner, *On the $\mathbb{Z}$-module structure of a free semialternative ring*, Communications in Algebra 27 (1999) 1951–1965.

[15] M. R. Bremner, *Identities for the ternary commutator*, Journal of Algebra 206 (1998) 615–623.

[16] M. R. Bremner, *Varieties of anticommutative n-ary algebras*, Journal of Algebra 191 (1997) 76–88.

Articles in refereed publications: submitted

[17] M. R. Bremner, *Jordan algebras arising from intermolecular recombination*, 12 pages, submitted 10 August 2005.

Other refereed contributions: conference proceedings

[18] M. R. Bremner, *DNA computing, insertion of words, and left-symmetric algebras*, Maple Conference 2005 Proceedings, Waterloo, July 17–20 (I. S. Kotsireas, editor), pages 229–242, Maplesoft, 2005 (ISBN 1-894511-85-9).

[19] M. R. Bremner and I. R. Hentzel, *Alternating triple systems with simple Lie algebras of derivations*, 28 pages, proofs received 3 October 2005, to appear in the Proceedings of NONAA-V (Fifth International Conference on Nonassociative Algebra and its Applications, Mexico, 2003), to be published by CRC Press, 2005.

[20] M. R. Bremner and **M. V. Kochetov** (translators), *Dniester Notebook: Unsolved Problems in the Theory of Rings and Modules*, fourth edition (1993), 56 pages, proofs received 7 October 2005, to appear in Proceedings of NONAA-V (Fifth International Conference on Nonassociative Algebra and its Applications, Mexico, 2003), to be published by CRC Press, 2005.

[21] M. R. Bremner and **Natalia Fomenko** (translators), *Some problems in the theory of rings that are nearly associative* by A. I. Shirshov (Uspekhi Mat. Nauk 13, 6 (1958) 3–20), 18 pages, proofs received 12 October 2005, to appear in Proceedings of NONAA-V (Fifth International Conference on Nonassociative Algebra and its Applications, Mexico, 2003), to be published by CRC Press, 2005.

NON-REFEREED CONTRIBUTIONS

[22] M. R. Bremner, Ten reviews in the last two years of journal articles for MathSciNet, the American Mathematical Society's *Mathematical Reviews on the Web*.

2.1 Collaborators and their affiliations

- Irvin R. Hentzel, Department of Mathematics, Iowa State University, Ames, Iowa, USA.
- Luiz A. Peresi, Instituto de Matemática e Estatística, Universidade de São Paulo, Brazil.

3 Other evidence of impact and contributions

I organized a Special Session on Nonassociative Algebra at the January 2004 meeting of the American Mathematical Society in Phoenix, Arizona. In the last 6 years I have given 5 invited seminar presentations at Iowa State University, and 4 at the University of São Paulo; and 8 talks at conferences on nonassociative algebra and computer algebra. I was invited to submit the lecture notes for my talk at the Second Conference on Lie and Jordan Algebras in Guarujá (Brazil) to a Brazilian journal; see [4]. In May 2005, I was invited by Maplesoft to submit a proposal for software to perform computations in nonassociative algebra. In 2003–04, I developed and taught a new graduate course on Computer Algebra. I received the Teaching Excellence Award for the Sciences from the College of Arts and Science of the University of Saskatchewan for the academic year 2004–2005. I am a member of the Board of Directors (expiring June 2007) and the Finance Committee (expiring December 2005) of the Canadian Mathematical Society.

4 Contributions to the training of highly qualified personnel

In the last six years I have supervised four summer undergraduate research students (**Mihilewicz**, **Ying Li**, **Lauren Bains**, **Richards**) and co-supervised a fifth (**Lee**). I have co-supervised two graduate students: one (name withheld) completed a Master's degree with another supervisor, and the other (**Lataianu**) is pursuing the Ph.D. degree in a topic suggested by another supervisor. I have supervised one post-doctoral fellow (**El Bachraoui**), and co-supervised another (**Kochetov**).

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Investir dans les gens, la découverte et l'innovation

SEND ONE ORIGINAL ONLY DO NOT PHOTOCOPY

APPENDIX A
Personal Data
(Form 100)

Complete this appendix (i) if you are an applicant or co-applicant applying for the first time; (ii) if you need to update information submitted with a previous application; or (iii) if you do not hold an appointment at a Canadian postsecondary institution. For updates, include only the revised information in addition to the date, your name and your PIN.

This information will be used by NSERC primarily to contact applicants and award holders. It may also be used to identify prospective reviewers and committee members, and to generate statistics. It will not be seen or used in the adjudication process.

Date: 2005/10/28

Family name	Given name	Initial(s) of all given names	Personal identification no. (PIN)
Bremner	Murray	MR	56851

Position and complete mailing address if your primary place of employment is not a Canadian postsecondary institution or if your current mailing address is temporary

If address is temporary, indicate:

Starting date

Leaving date

Telephone number	Facsimile number	E-mail address
(306) 966-6122	(306) 966-6086	bremner@math.usask.ca

Telephone number (alternate): (306) 966-6081

Give an alternate telephone number only if you can be reached at that number during business hours.

Gender (completion optional): [X] Male [ ] Female

LANGUAGE CAPABILITY

	Read	Write	Speak
English	X	X	X
French	X		

I wish to receive my correspondence: in English [X] in French [ ]

AREA(S) OF EXPERTISE

Provide a maximum of 10 key words that describe your area(s) of expertise. Use commas to separate them. If you have expertise with particular instruments and techniques, specify which one(s).

Nonassociative algebra, Polynomial identities, Computational algebra, Algorithms for linear algebra, Maple programming, Representation theory, Lie algebras, Quantum groups

Research subject code(s)

Primary: 2908

Secondary: 2714

Canada

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Appendix D (Form 100)
Consent to Provide Limited Personal Information About
Highly Qualified Personnel (HQP) to NSERC

NSERC applicants are required to describe their contributions to the training or supervision of highly qualified personnel (HQP) by providing certain details about the individuals they have trained or supervised during the six years prior to their current application. HQP information must be entered on the Personal Data Form (Form 100). This information includes the trainee's name, type of HQP training (e.g., undergraduate, master's, technical etc.) and status (completed, in-progress, incomplete), years supervised or co-supervised, title of the project or thesis, and the individual's present position.

Based on the federal *Privacy Act* rules governing the collection of personal information, applicants are asked to obtain consent from the individuals they have supervised before providing personal data about them to NSERC. In seeking this consent, the NSERC applicant must inform these individuals what data will be supplied, and assure them that it will only be used by NSERC for the purpose of assessing the applicant's contribution to HQP training. To reduce seeking consent for multiple applications, applicants will only need to seek consent one time for a six-year period. If the trainee provides consent by e-mail, the response must include confirmation that they have read and agree to the text of the consent form.

When consent cannot be obtained, applicants are asked to not provide names, or other combinations of data, that would identify those supervised. However, they may still provide the type of HQP training and status, years supervised or co-supervised, a general description of the project or thesis, and a general indication of the individual's present position if known.

An example of entering HQP information on Form 100 (with and without consent):

Name	Type of HQP Training and Status	Years Supervised or Co-supervised	Title of Project or Thesis	Present Position
Consent Received from Marie Roy				
Roy, Marie	Undergraduate (Completed)	Supervised 1994 - 1997	Isotope geochemistry in petroleum engineering	V-P (Research), Earth Analytics Inc., Calgary, Alberta
Consent Not Obtained from Marie Roy				
(name withheld)	Undergraduate (Completed)	Supervised 1994 - 1997	Isotope geochemistry	research executive in petroleum industry - western Canada

Consent Form

Name of Trainee

Applicant Information

Name Bremner, Murray MR

Department Mathematics and Statistics

Postsecondary Institution Saskatchewan

I hereby allow the above-named applicant to include limited personal data about me in grant applications submitted for consideration to NSERC for the next six years. This limited data will only include my name, type of HQP training and status, years supervised or co-supervised, title of the project or thesis and, to the best of the applicant's knowledge, my position title and company or organization at the time the application is submitted. I understand that NSERC will protect this data in accordance with the *Privacy Act*, and that it will only be used in processes that assess the applicant's contributions to the training of highly qualified personnel (HQP), including confidential peer review.

Trainee's signature

Date

Note: This form must be retained by the applicant and made available to NSERC upon request.