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It is also common, (and useful in naval, aeronautical, and military situations) to specify them in polar form. The point $P = (r, \theta)$ lies r units from the origin O and the line OP makes an angle θ with the positive x-axis. The polar form is not unique: every point has an infinite number of polar coordinates.

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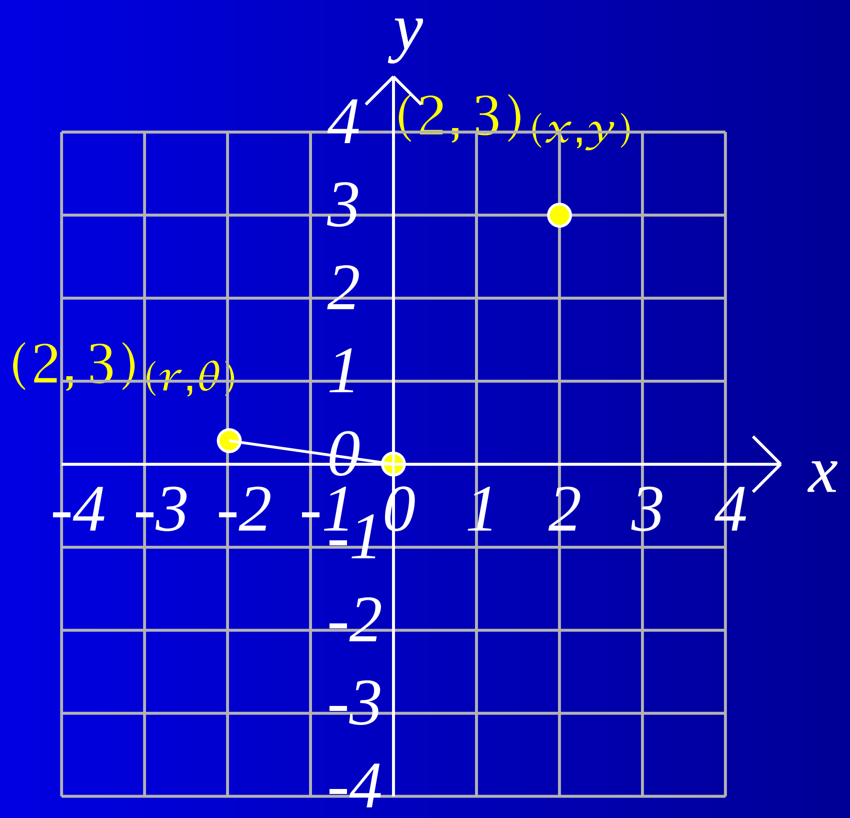

We easily generalize this concept to points $P = (r, \theta)_{(r,\theta)}$ where r is negative by taking P in the negative direction along the line which makes the angle θ with the positive x-axis. Although negative r values seldom occur in real-life, they quite naturally occur in polar equations: equations involving r and θ, such as $r = f(\theta)$.

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See the Java applet with no curve selected.

The curves displayable using the Polar Java applets have the polar equations:

Astroid: $r(\theta) = a\sqrt{\cos^4\theta - \cos^2\theta\sin^2\theta + \sin^4\theta}$

Daisy: $r(\theta) = a + b\sin(c(\theta - d))$

Generalized conics: $r(\theta) = a + \dfrac{b}{1 + c\sin(d(\theta - e))}$

Lemniscate: $r(\theta) = a + b\sqrt{|\cos(2c(\theta - d))|}$

Spiral: $r(\theta) = a + b\theta + c\theta^2 + d\theta^3 + e\theta^4$

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$\dfrac{dx}{d\theta}$ and $\dfrac{dy}{d\theta}$ are 0 or undefined.

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which we summarize:

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or using vector and matrix notation:

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These can be used to find the slope and concavity of the curve:

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$$\frac{d^2y}{dx^2}=\frac{\frac{dx}{d\theta}\frac{d^2y}{d\theta^2}-\frac{d^2x}{d\theta^2}\frac{dy}{d\theta}}{\left(\frac{dx}{d\theta}\right)^3}=$$

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$$\frac{r(\theta)^2 - \ddot{r}(\theta)r(\theta) + 2\dot{r}(\theta)^2}{[-r(\theta)\sin\theta + \dot{r}(\theta)\cos\theta]^3}$$

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(p671,Stewart4) Find the area enclosed by one loop of the four-leaved rose $r = \cos 2\theta$.

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$$\int_0^{\frac{\pi}{4}} \left(\frac{1 + \cos 4\theta}{2} \right) d\theta = \left[\frac{\theta}{2} + \frac{\sin 4\theta}{8} \right] \Bigg|_0^{\frac{\pi}{4}} = \frac{\pi}{8}$$

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Example:(Bob Adams,8.6-Problem 13,p.515)

Find the length of the curve $r = e^{a\theta}, -\pi \le \theta \le \pi$.

Solution: We have $\dot{r} = ae^{a\theta}$, so

$$\dot{r}^2 + r^2 = \left(ae^{a\theta}\right)^2 + \left(ae^{a\theta}\right)^2 = (a^2+1)e^{2a\theta} =$$

and so $\sqrt{\dot{r}^2 + r^2} = \sqrt{a^2+1}e^{2a\theta}$, and thus

$$L = \int_{-\pi}^{\pi} \sqrt{a^2+1}e^{a\theta}d\theta = \sqrt{a^2+1}\int_{-\pi}^{\pi} e^{a\theta}d\theta =$$

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Example:(Bob Adams,8.6-Problem 14,p.515)

Find the length of the curve $r = a\theta,\ 0 \le \theta \le 2\pi$.

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$$\frac{a}{2}\left(\sec u \tan u + \ln|\sec u + \tan u|\right)\Big|_{u=0}^{u=\arctan 2\pi} =$$

$$\frac{a}{2}\left(\sqrt{1+4\pi^2}(2\pi)+\ln|\sqrt{1+4\pi^2}+2\pi|\right)=$$

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$$\frac{a}{2}\left(2\pi\sqrt{1+4\pi^2}+\ln\left|\sqrt{1+4\pi^2}+2\pi\right|\right)$$

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