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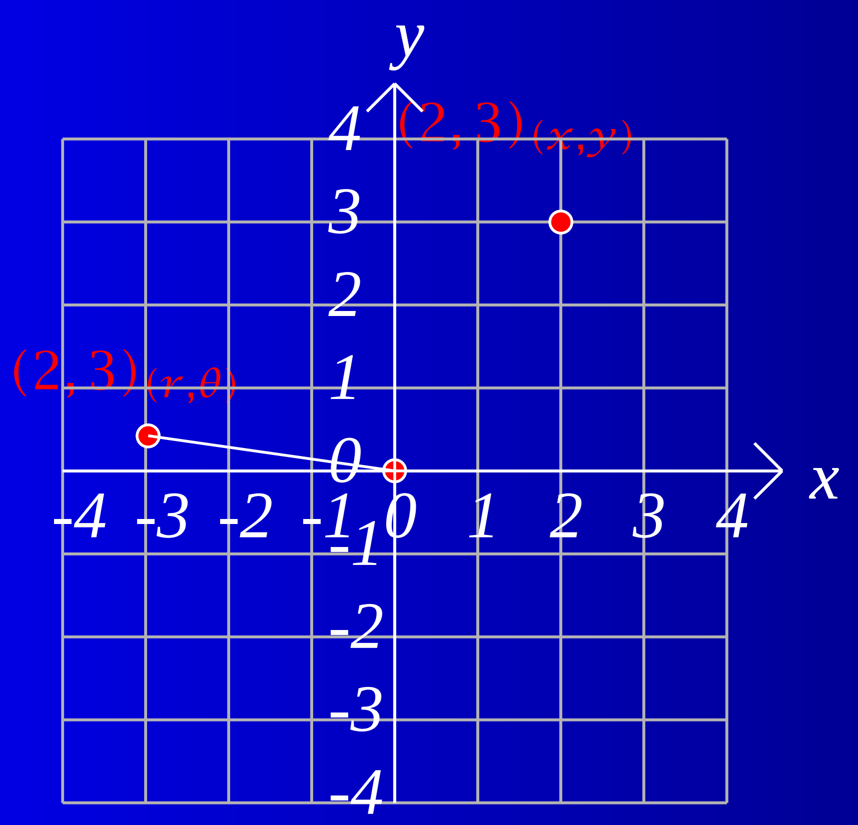

We easily generalize this concept to points $P = (r, \theta)_{(r,\theta)}$ where $r$ is negative by taking $P$ in the negative direction along the line which makes the angle $\theta$ with the positive $x$-axis. Although negative $r$ values seldom occur in real-life, they quite naturally occur in polar equations: equations involving $r$ and $\theta$, such as $r = f(\theta)$.

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See the Java applet with no curve selected.

The curves displayable using the Polar Java applets have the polar equations:

Astroid: $r(\theta) = a\sqrt{\cos^4\theta - \cos^2\theta\sin^2\theta + \sin^4\theta}$

Daisy: $r(\theta) = a + b\sin(c(\theta - d))$

Generalized conics: $r(\theta) = a + \dfrac{b}{1 + c\sin(d(\theta - e))}$

Lemniscate: $r(\theta) = a + b\sqrt{|\cos(2c(\theta - d))|}$

Spiral: $r(\theta) = a + b\theta + c\theta^2 + d\theta^3 + e\theta^4$

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$\dfrac{dx}{d\theta}$ and $\dfrac{dy}{d\theta}$ are 0 or undefined.

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These can be used to find the slope and concavity of the curve:

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(p671,Stewart4) Find the area enclosed by one loop of the four-leaved rose $r = \cos 2\theta$.

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$$\int_0^{\frac{\pi}{4}} \left(\frac{1+\cos 4\theta}{2}\right) d\theta = \left[\frac{\theta}{2} + \frac{\sin 4\theta}{8}\right]\Big|_0^{\frac{\pi}{4}} = \frac{\pi}{8}$$

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Solution: Solving $2n\theta = \frac{\pi}{2}$ for $\theta$, we get $\theta = \frac{\pi}{4n}$, so one loop is traced out as $\theta$ runs from $-\frac{\pi}{4n}$ to $\frac{\pi}{4n}$.

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Solution: Solving $2n\theta = \frac{\pi}{2}$ for $\theta$, we get $\theta = \frac{\pi}{4n}$, so one loop is traced out as $\theta$ runs from $-\frac{\pi}{4n}$ to $\frac{\pi}{4n}$. Using symmetry, we calculate twice the area of the one-half of the loop traced as $0 \leq \theta \leq \frac{\pi}{4n}$.

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which cannot be evaluated in closed form.

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$$\frac{8}{3}\left[\left(1 + \left(\frac{\pi}{2}\right)^2\right)^{\frac{3}{2}} - 1\right] =$$

$$\frac{8}{3}\left[\frac{(4+\pi^2)^{\frac{3}{2}}}{8} - 1\right] = \frac{(4+\pi^2)^{\frac{3}{2}} - 8}{3}$$

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