THE BOOLEAN SPACE OF $\mathbb{R}$-PLACES
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An $\mathbb{R}$-place of a formally real field $K$ is a place $\xi : K \to \mathbb{R} \cup \{\infty\}$. The set of all $\mathbb{R}$-places of the field $K$ is denoted by $M(K)$. Every $\mathbb{R}$-place of $K$ is connected with some subset of the space $X(K)$ of orderings of the field $K$. Namely, if $\xi$ is an $\mathbb{R}$-place, then there exists an ordering $P$ such that the set

$$A(P) := \{ a \in K : \exists q \in \mathbb{Q}^+ (q \pm a \in P) \}$$

is the valuation ring of $\xi$. We say that $P$ determines $\xi$ in this case. Any ordering $P$ of the field $K$ determines exactly one $\mathbb{R}$-place.

The above described correspondence between orderings and $\mathbb{R}$-places defines a surjective map

$$\lambda_K : X(K) \to M(K),$$

which, in turn, allows us to equip $M(K)$ with the quotient topology inherited from $X(K)$. $M(K)$ is a Hausdorff space. It is also compact as a continuous image of a compact space. But the problem

Which compact, Hausdorff spaces occur as spaces of real places?

is still open.

We prove that every Boolean space is a space of $\mathbb{R}$-places of some formally real field $K$. 