University of Saskatchewan
Department of Mathematics & Statistics

Instructor:	MATHEMATICS 264.3 (01)	Time: 3 hours
G. Griffith	FINAL EXAMINATION	December 21, 2000

Calculators are not allowed.
Closed Book
There is no penalty for an incorrect response.

Questions 1–31 are worth one mark each.
Questions 32–41 are worth two marks each.

The choices for all questions are

(a) -3 (b) -2 (c) -1 (d) 0 (e) 1

(f) 2 (g) 3 (h) 4 (i) 5 (j) 6

Consider the matrix $E = \begin{pmatrix} 2 & -3 \\ 3 & -4 \end{pmatrix}$. Perform the following elementary row operations on E in this order in order to transform E into its reduced row echelon form.

(i) Subtract row 1 from row 2.

(ii) Interchange the rows.

(iii) Add -2 times row 1 to row 2.

(iv) Multiply row 2 by -1.

(v) Add row 2 to row 1.

The matrix will now be in reduced row echelon form.

Let P, Q, R, S, T be the elementary matrices which correspond to operations (i) (ii) (iii) (iv) (v) respectively.
Let $X = (x_{ij}) = RQ$, $Y = (y_{ij}) = SR$, $Z = (z_{ij}) = TS$.
Then

1. $x_{11} =$ 2. $x_{22} =$ 3. $y_{21} =$

4. $y_{12} =$ 5. $z_{12} =$ 6. $z_{22} =$

$$\text{Let} \quad A = \begin{pmatrix} 2 & -1 & 1 \\ 3 & -1 & -1 \\ 1 & 2 & -3 \end{pmatrix} .$$

Let $M = (m_{ij})$ be the matrix of minors and let $B = (b_{ij}) = adj\,.\,A$. Then

7. $m_{11} =$ 8. $m_{21} =$ 9. $m_{31} =$ 10. $m_{33} =$

11. $b_{11} =$ 12. $b_{12} =$ 13. $b_{23} =$ 14. $b_{33} =$

Use this information to solve the linear system

$$\begin{aligned} 2x - y + z &= 2 \\ 3x - y - z &= -4 \\ x + 2y - 3z &= 2 \end{aligned} .$$

(Note that the matrix of coefficients is A.)

Then 15. $x =$ 16. $y =$ 17. $z =$

The line $\dfrac{x+8}{10} = \dfrac{y-3}{3} = \dfrac{z-8}{-4}$ intersects the plane $x+2z = 10$ at $P = (p_1, p_2, p_3)$.

Let θ be the acute angle which the line makes with the normal to the plane. (Recall that the cosine of an acute angle is positive.)

Then 18. $p_1 =$ 19. $p_2 =$ 20. $p_3 =$ 21. $25\cos\theta =$

The lines

$$\frac{x+6}{2} = \frac{y-2}{1} = \frac{z-8}{-1}$$

$$\text{and} \quad \frac{x+1}{1} = \frac{y}{2} = \frac{z-1}{1}$$

intersect at $Q = (q_1, q_2, q_3)$. The plane which they determine has equation $1x + by + cz = d$.

Then 22. $q_1 =$ 23. $q_2 =$ 24. $q_3 =$

and 25. $b =$ 26. $c =$ 27. $d =$

28. If ϕ is the acute angle between the lines, then $2\cos\phi =$

If $ax + by + cz = -6$ is an equation of the plane which contains the line $\dfrac{x-1}{2} = \dfrac{y}{3} = \dfrac{z-2}{1}$ and which also passes through the point $(2, 6, 4)$ then

29. $a =$ 30. $b =$ 31. $c =$

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32. If the matrices

$$\begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 2 \\ -1 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 3 & 2 \end{pmatrix}, \begin{pmatrix} 0 & -7 \\ x & -4 \end{pmatrix}$$

are linearly dependent, then $x =$

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33. If $(\,y \quad 14 \quad 19\,)$ is a vector in the span of $S = \{(\,2 \quad 6 \quad 1\,), (\,-1 \quad -2 \quad 3\,)\}$ then $y =$

34. Consider the polynomials

$$p_1 = 3 - 2x + x^2, \quad p_2 = 2 + 3x + 2x^2, \quad p_3 = a - 18x - 5x^2 \ .$$

If they fail to span P_2, then $a =$

Compute the rank, R and the nullity, N of the matrix

$$\begin{pmatrix} 1 & 2 & 4 & -1 & 2 \\ 2 & 1 & -1 & 3 & 1 \\ 1 & 0 & -2 & 0 & 0 \\ 1 & 1 & 1 & -4 & 1 \end{pmatrix}$$

then 35. $R =$ 36. $N =$

Consider the matrix $A = \begin{pmatrix} 6 & -2 \\ 5 & -1 \end{pmatrix}$

Let λ_1, λ_2 be its eigenvalues with $\lambda_1 > \lambda_2$.

If $X_1 = \begin{pmatrix} p \\ 1 \end{pmatrix}$, $X_2 = \begin{pmatrix} q \\ 5 \end{pmatrix}$ are eigenvectors which correspond to λ_1, λ_2 respectively, then

37. $\lambda_1 =$ 38. $\lambda_2 =$ 39. $p =$ 40. $q =$

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41. Consider the matrix $B = \begin{pmatrix} -1 & 2 & 2 \\ 1 & -5 & -1 \\ 5 & 0 & 2 \end{pmatrix}$. One of its eigenvalues is an integer, k.

Then $k =$

** THE END **