

UNIVERSITY OF SASKATCHEWAN  
Department of Mathematics and Statistics

Mathematics 238.3 (01)  
Final Examination

Time: 3 hours  
December 19, 2000

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**Closed Book**

Marks:

1. Determine whether the following series converge or diverge. Clearly indicate what test you are using and show all your work.

[4] (a)  $\sum_{n=1}^{\infty} \frac{\cos n\pi}{2n+3}$

[4] (b)  $\sum_{n=1}^{\infty} \frac{n!+1}{(n+1)!}$

[4] (c)  $\sum_{n=2}^{\infty} \frac{1}{(\ln(\ln n))^2 n \ln n}$

- [4] 2. (a) A rubber ball is dropped from a height of 4 m and bounces back to half its height after each fall. If it continues to bounce indefinitely, find the total distance it travels.

- [6] (b) Find the sum of the following series.

(i)  $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n} (2n)!}$

(ii)  $3 + \frac{9}{2!} + \frac{27}{3!} + \frac{81}{4!} + \dots$

(iii)  $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{4^{2n+1} (2n+1)!}$

- [7] 3. Find the interval of convergence of the power series  $\sum_{n=1}^{\infty} \frac{e^n (4-x)^n}{n}$  and state the values of  $x$  (if any) for which the series converges conditionally.

- [7] 4. Find the Maclaurin series representation for  $f(x) = \frac{2}{\sqrt{4+x^2}}$  and state the radius of convergence.

- [6] 5. Use series to approximate  $\int_0^{.5} \cos(x^2) dx$  correct to 3 decimal places.

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6. Prove or disprove the following statements.

- [2] (a) If the sequence  $\{|a_n|\}$  converges, then  $\{a_n\}$  converges.
- [4] (b) If  $\sum_{n=1}^{\infty} a_n$  and  $\sum_{n=1}^{\infty} b_n$  are convergent series with positive terms, then  $\sum_{n=1}^{\infty} a_n b_n$  is convergent.

7. Solve the following differential equations.

- [4] (a)  $x^2 y'' + 5xy' + 13y = 0, \quad x > 0$
- [6] (b)  $(y - x + xy \cot x)dx + xdy = 0$
- [6] (c)  $y'' - 2y' = x + e^{2x}$
- [6] (d)  $3y^2 y' + y^3 = e^{-x}$
- [6] (e)  $y'' - 3y' + 2y = \sec^2(e^{-x})$
- [6] (f)  $(x - y \ln y + y \ln x)dx + x(\ln y - \ln x)dy = 0$

- [8] 8. A spring fixed at its upper end is stretched 6 inches by a 10 - pound weight attached at its lower end. The spring-mass system is suspended in a viscous fluid so that the system is subjected to a damping force (in pounds) with damping constant equal to 5. Find the position of the mass at time  $t$  sec if the weight is drawn down an additional 4 inches and released with a velocity of 0 ft per sec. Include a diagram which indicates the motion of the system.

- [4] 9. (a) State conditions which imply that an initial-value problem for a first-order differential equation has a unique solution near the initial point.
- [6] (b) Give an example of an initial-value problem having non-unique solutions: What conditions in part (a) does your example fail to satisfy?

[100] Total

\*\* The End \*\*