

University of Saskatchewan  
Department of Mathematics and Statistics

Time: 3 hours  
9:00 a.m. to 12:00 p.m.

MATHEMATICS 226.3 (02)  
FINAL EXAMINATION

April 16, 2001  
Instructor: J. R. Martin

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**CLOSED BOOK**

Marks:

1. Find the limit, if it exists, of the given sequence

[4] (a)  $\{ n - \sqrt{n^2 + 3n} \}$

[4] (b)  $\{ \sqrt[n]{e^n + 3^n} \}$

2. If the given series converges, find its sum. If it diverges, indicate why.

[4] (a)  $\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1}$

[4] (b)  $-\ln 3 + \frac{(\ln 3)^2}{2!} - \frac{(\ln 3)^3}{3!} + \frac{(\ln 3)^4}{4!} - \dots$

3. Determine whether the following series converge or diverge. Clearly indicate what test you are using and show all your work.

[4] (a)  $\sum_{n=1}^{\infty} \frac{1}{n + \ln(n^4)}$

[4] (b)  $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$

[4] (c)  $\sum_{n=1}^{\infty} \frac{n^{100} 2^n}{\sqrt{n!}}$

[4] (d)  $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{\ln(2n)}}$

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- [6] 4. Determine the values of  $x$  for which the series  $\sum_{n=1}^{\infty} \frac{(1 + \frac{1}{x})^n}{n}$  converges absolutely, converges conditionally or diverges.
- [8] 5. (a) Find the MacLaurin series representation for  $\int_0^t \frac{\ln(1+x)}{x} dx$  and give the interval of convergence.
- [4] (b) Estimate  $\int_0^{.5} \frac{\ln(1+x)}{x} dx$  correct to 3 decimal places.
6. Solve the following differential equations
- [4] (a)  $y'(x^2 + 2x + 1) + \sqrt{1 - y^2} = 0$
- [6] (b)  $y'(\cos^3 y - x \sin y) + \cos y = 0$
- [6] (c)  $y'' - 2y' + y = \frac{e^x}{1 + x^2}$
- [6] (d)  $y'(x \ln x) + 3y - 6 = 0$
- [6] (e)  $y'(xy e^{\frac{y}{x}}) = y^2 e^{\frac{y}{x}} + x^2$
- [6] (f)  $y'' + y = \sin^2 x$
- [8] 7. Use power series to solve the differential equation  $y'' - xy = 0$ .
- [8] 8. A spring with a mass of 10 kg has damping constant  $\frac{1}{2}$  and vibrates freely. What is the spring constant if the period of motion is 8 sec.?

[100] Total

\*\*\* The End \*\*\*